Differentiation practice test 3: representations and models

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

An original progressively demanding chapter paper. Show analytical working; a scientific calculator may be used for numerical values. Diagrams and data needed for questions are supplied.

A scientific calculator may be used.

1. A student claims the derivative of $(x^2+1)\sin x$ is $2x\cos x$. [5 marks]

(a) Explain the error and give the correct derivative. [3 marks]

(b) Find the tangent at the origin. [2 marks]

2. Let $f(x) = 1/(2x-1)^2$. [5 marks]

(a) Find its first derivative and domain. [3 marks]

(b) Find its second derivative. [2 marks]

3. Consider $g(x) = e^{x^2-4x}$. [6 marks]

(a) Find and classify its stationary point. [3 marks]

(b) Show the curve is concave up everywhere. [3 marks]

4. Let $h(x) = \sin x + \cos x$ for $0 \leq x \leq 2\pi$. [6 marks]

(a) Find and classify its stationary points. [4 marks]

(b) Are these also the absolute extrema on the interval? [2 marks]

5. Analyse $F(x) = x^4 - 4x^3 + 1$. [7 marks]

(a) Find and classify stationary points. [4 marks]

(b) Find all inflection points. [3 marks]

6. A model has population $P(t) = 100e^{0.2t}$, with time in hours. Rounded observations are supplied. [6 marks]

t (h)	P (organisms)
0	100
1	122.14
2	149.18
3	182.21

(a) Use the observations at 0 and 2 to estimate the instantaneous rate near 1 hour. [3 marks]

(b) Compare with the exact model rate at 1 hour. [3 marks]

7. The cubic $f(x) = x^3 + ax^2 + bx + c$ is stationary at $x = -1$ and $x = 1$, and $f(1) = -1$. [6 marks]

(a) Find a and b. [3 marks]

(b) Find c and the other stationary height. [3 marks]

8. An open box is made by cutting squares of side x from a 20 cm square and folding up the sides. [6 marks]

(a) Give the volume function and feasible domain. [2 marks]

(b) Find the maximum volume. [4 marks]

9. A point on the line $y = 2x + 1$ is to be as close as possible to the origin. [6 marks]

(a) Write a function for squared distance. [2 marks]

(b) Find the point and its distance. [4 marks]

10. A business earns $R(q) = q(50 - q)$ dollars and pays $C(q) = 10q + 100$ dollars for $0 \leq q \leq 50$ units. [7 marks]

(a) Form the profit function. [2 marks]

(b) Find maximum profit and the required output. [3 marks]

(c) Why is maximising revenue alone not enough? [2 marks]