Differentiation practice test 4: connected methods

Name: _____ Date: _____

10 questions | 61 marks | 75 minutes

Original chapter assessment with linked reasoning and applications. Show working, state relevant domains and retain exact values until a numerical answer is requested.

A scientific calculator may be used.

1. Let $f(x) = ax^3 + bx^2$, where $f'(1) = 0$ and $f''(1) = 6$. [5 marks]

(a) Find a and b. [3 marks]

(b) Classify the stationary point at $x = 1$. [2 marks]

2. Consider $g(x) = (x-1)^2 e^x$. [7 marks]

(a) Differentiate and factorise. [3 marks]

(b) Find and classify both stationary points. [4 marks]

3. Let $h(x) = \sin x/(1+\cos x)$. [6 marks]

(a) Find its derivative and state the excluded inputs. [4 marks]

(b) Find the tangent at zero. [2 marks]

4. For $f(x) = \sin(2x)\cos(2x)$, all inputs are radians. [5 marks]

(a) Use the product rule to differentiate. [3 marks]

(b) Given the equivalent form $f(x) = \frac{1}{2}\sin(4x)$, find $f''(x)$. [2 marks]

5. A smooth function satisfies $f''(x) = (x-1)^2(x+2)$ for all real x. [7 marks]

(a) Find all inputs at which an inflection could occur. [2 marks]

(b) Decide which candidate is a true inflection input. [3 marks]

(c) Can its inflection height be found from this information alone? [2 marks]

6. A population follows $P(t) = 600/(1 + 5e^{-0.4t})$. Time is in days. [8 marks]

(a) Find the initial population and initial rate. [4 marks]

(b) The derivative can also be written as $P' = 0.4P(1 - P/600)$. At what population is the growth rate greatest, and what is that rate? [4 marks]

7. Let $f(x) = x^3 - 3x$. [5 marks]

(a) Find inputs where the tangent is parallel to $y = 3x + 7$. [2 marks]

(b) Find both tangent equations. [3 marks]

8. A rectangle in the first quadrant has one vertex at the origin and its upper-right vertex on $y = 12 - x^2$. Its sides are parallel to the axes. [6 marks]

(a) Form an area function and state a feasible domain. [2 marks]

(b) Find its maximum area and dimensions. [4 marks]

9. A particle has position $s(t) = 1 + \sin(2t)$ metres for $0 \leq t \leq \pi$ seconds. [6 marks]

(a) Find its reversal times. [3 marks]

(b) Find total distance and displacement. [3 marks]

10. Total production cost is $C(q) = 0.5q^2 + 200$ dollars for $q > 0$ units. [6 marks]

(a) Form an expression for average cost per unit. [2 marks]

(b) Minimise average cost, first treating output as continuous, then checking whole units. [4 marks]