Differentiation practice test 6: synthesis and applications

Name: _____ Date: _____

10 questions | 76 marks | 75 minutes

Original chapter assessment with linked reasoning and applications. Show working, state relevant domains and retain exact values until a numerical answer is requested.

A scientific calculator may be used.

1. Differentiate efficiently, stating any excluded inputs. [5 marks]

(a) $f(x) = (x^2 - 1)/(x + 1)$. [2 marks]

(b) $g(x) = (x^2 + 1)^3 e^{-x}$. [3 marks]

2. A temperature model is $T(t) = 18 + 6\sin[\pi(t-4)/12]$ degrees Celsius, for $0 \le t \le 24$ hours. [9 marks]

(a) Find the maximum and minimum temperatures and their times. [3 marks]

(b) Find when warming is fastest, and the rate. [3 marks]

(c) Find both times when the temperature is 20 degrees Celsius. [3 marks]

3. Let $f(x) = x^3 - 3ax$, with $a > 0$. [7 marks]

(a) Find stationary coordinates in terms of a. [3 marks]

(b) Classify them. [2 marks]

(c) Their vertical separation is 32. Find a. [2 marks]

4. A cooling object follows $T(t) = 20 + 80e^{-kt}$ degrees Celsius, with t in minutes. After 5 minutes its temperature is 60 degrees Celsius. [8 marks]

(a) Find k exactly. [3 marks]

(b) Find the initial rate of temperature change. [3 marks]

(c) Find when its temperature reaches 30 degrees Celsius. [2 marks]

5. The curve $y = (x^2 + ax + b)/(x - 1)$ has oblique asymptote $y = x + 3$ and y-intercept -4. [9 marks]

(a) Find a and b. [3 marks]

(b) Find stationary coordinates. [4 marks]

(c) Classify them. [2 marks]

6. A closed cylindrical tank has volume 200π cubic metres. Material for its top and base costs twice as much per square metre as material for its curved side. [7 marks]

(a) Give a cost function proportional to the actual cost, in terms of radius r. [3 marks]

(b) Find the optimum relation between height and radius. [4 marks]

7. A particle has $s(t) = t^4/4 - 2t^3 + 9t^2/2 - 4t$ metres, $0 \leq t \leq 5$ seconds. [9 marks]

(a) Factorise the velocity and find its rest times. [3 marks]

(b) Which rest time is a reversal? [2 marks]

(c) Find displacement and total distance. [4 marks]

8. A rectangle in the first quadrant has its upper-right corner on $y = 12e^{-x}$ and its lower-left corner at the origin. Its sides are parallel to the coordinate axes. [6 marks]

(a) Form the area function and find its derivative. [3 marks]

(b) Find and justify its maximum area. [3 marks]

9. A smooth function satisfies $f'(x) = (x^2 - 1)e^{-x}$. [8 marks]

(a) Find and classify the stationary inputs. [3 marks]

(b) Find the inputs where concavity changes. [3 marks]

(c) Can the stationary heights be determined from the supplied information alone? [2 marks]

10. A product has demand price $p(q) = 80 - 0.5q$ dollars per unit and cost $C(q) = 20q + 200$ dollars, where $0 \leq q \leq 160$. [8 marks]

(a) Form profit as a function of output. [2 marks]

(b) Find the profit-maximising output, price and profit. [4 marks]

(c) A student proposes 80 units by maximising revenue. Explain why that does not maximise profit. [2 marks]