Logarithmic graphs, domains and transformations

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. State the domain and asymptote of $y = \log_2(x - 3)$. [2 marks]

The input $x - 3$ must be positive; its zero gives the asymptote.

Answer: $x > 3$; $x = 3$

2. Find the x-intercept of $y = \ln x - 2$. [2 marks]

$\ln x = 2$ gives $x = e^2$.

Answer: $(e^2, 0)$

3. Find the domain and y-intercept of $y = \ln(6 - 2x)$. [3 marks]

$6 - 2x > 0$ gives $x < 3$; substitute $x = 0$.

Answer: $x < 3$; $(0, \ln 6)$

4. Describe the transformation from $y = \ln x$ to $y = -2\ln x + 3$. [2 marks]

The multiplier -2 changes output values; the $+3$ is applied afterwards.

Answer: Reflection in the x-axis, vertical scale factor 2, then translation up 3.

5. Find the inverse of $f(x) = \ln(x + 2) - 1$ and its domain. [3 marks]

$y + 1 = \ln(x + 2)$ gives $x = e^{y+1} - 2$. The original range is all real numbers.

Answer: $f^{-1}(x) = e^{x+1} - 2$, $x \in \mathbb{R}$

6. Sketch $y = \ln x$ and $y = \ln(2x)$ on the same axes using exact anchor points. State their vertical separation. [3 marks]

For $x > 0$, $\ln(2x) = \ln x + \ln 2$. Anchor points include $(1, 0)$ and $(1, \ln 2)$.

Answer: The second graph is $\ln 2$ units above the first; both have asymptote $x = 0$.