Logarithmic functions - Practice test 1

10 questions | 50 marks | 70 minutes

A balanced paper moving from inverse relationships to equations, graphs, derivatives and applications.

Equivalent correct methods earn the corresponding marks.

1. Use logarithms as exponents. [4 marks]

(a) Evaluate $\log_2(1/32)$ and explain the sign. [2 marks]

$2^{-5} = 1/32$.

Answer: -5; a negative exponent gives a positive reciprocal.

(b) Solve $\log_3 x = 5/2$ exactly. [2 marks]

$x = 3^{5/2} = 9\sqrt{3}$.

Answer: $x = 9\sqrt{3}$

2. Apply laws of logarithms. [4 marks]

(a) For $x, y > 0$, condense $2\ln x + \ln 3 - \ln y$. [2 marks]

The power law gives $\ln(x^2)$; product and quotient laws combine the inputs.

Answer: $\ln(3x^2/y)$

(b) Give a numerical counterexample to $\ln(a+b) = \ln a + \ln b$. [2 marks]

The right side is $\ln 1 + \ln 1 = 0$, whereas $\ln 2 > 0$.

Answer: For $a = b = 1$, $\ln 2 \neq 0$.

3. An unfamiliar logarithm base is needed. [4 marks]

(a) Show that $\log_7 20 = \ln 20/\ln 7$. [2 marks]

Set $u = \log_7 20$ and take natural logarithms.

Answer: $7^u = 20 \Rightarrow u\ln 7 = \ln 20$.

(b) Evaluate $\log_7 20$ to three decimal places. [2 marks]

$\ln 20/\ln 7 \approx 1.539502$; to three decimal places this is 1.540.

Answer: 1.540

4. Solve equations while keeping the original domains. [5 marks]

(a) Solve $\ln(x-2) = \ln 7$. [2 marks]

The domain is $x > 2$. Equal logarithms have equal positive inputs, so $x - 2 = 7$.

Answer: $x = 9$

(b) Solve $\ln(x-1) + \ln(x+1) = \ln 15$. [3 marks]

The domain is $x > 1$. Then $x^2 - 1 = 15$ gives $x = \pm 4$; only 4 is valid.

Answer: $x = 4$

5. Solve exponential equations exactly. [5 marks]

(a) Solve $4 \cdot 3^{2x} = 28$. [2 marks]

$3^{2x} = 7$, then $2x\ln 3 = \ln 7$.

Answer: $x = \ln 7/(2\ln 3)$

(b) Find all real solutions of $e^{2x} - 7e^x + 12 = 0$. [3 marks]

Put $u = e^x > 0$. $(u-3)(u-4) = 0$; both roots are positive.

Answer: $x = \ln 3$ or $x = \ln 4$

6. Work with natural logarithms. [5 marks]

(a) Simplify $\ln(5e^2) - \ln 5$ exactly. [2 marks]

Use the quotient law: $\ln(e^2) = 2$.

Answer: 2

(b) Solve $2\ln x = \ln(2x + 3)$. [3 marks]

Domain $x > 0$. The equation becomes $x^2 - 2x - 3 = 0$, giving $3, -1$. Reject -1.

Answer: $x = 3$

7. Let $f(x) = \ln(x - 2) + 3$. [6 marks]

(a) State the domain and vertical asymptote. [2 marks]

The logarithm input must be positive and tends to zero at $x = 2$.

Answer: $x > 2$; asymptote $x = 2$.

(b) Find the x-intercept and the inverse function. [4 marks]

At $y = 0$, $\ln(x - 2) = -3$. For the inverse solve $y - 3 = \ln(x - 2)$ for x, then swap names.

Answer: $(2 + e^{-3}, 0)$; $f^{-1}(x) = 2 + e^{x-3}$.

8. Consider $y = \ln(5 + 2x)$. [5 marks]

(a) Differentiate and state the real domain. [2 marks]

Apply input derivative divided by input; require $5 + 2x > 0$.

Answer: $y' = 2/(5 + 2x)$; $x > -5/2$.

(b) Find the tangent line at x = 0. [3 marks]

The point is $(0, \ln 5)$ and the gradient is 2/5.

Answer: $y = \ln 5 + \frac{2}{5}x$.

9. A training score follows $S(t) = 20 + 8\ln(t+1)$ for $t \geq 0$ days. [6 marks]

(a) Find when $S(t) = 36$. Give an exact answer and then three significant figures. [3 marks]

$8\ln(t+1) = 16$, so $t + 1 = e^2$.

Answer: $t = e^2 - 1 \approx 6.39$ days.

(b) Find the instantaneous rate at day 3 and explain its units. [3 marks]

$S'(t) = 8/(t+1)$, so $S'(3) = 2$.

Answer: 2 score units per day.

10. A supplied intensity scale is $L = 10\log_{10}(I/I_0)$, where $I, I_0 > 0$. [6 marks]

(a) Find the intensity ratio associated with a 20-unit rise in L. [3 marks]

$20 = 10\log_{10}(I_2/I_1)$; exponentiating gives 10^2.

Answer: $I_2/I_1 = 100$.

(b) Explain why the model cannot accept I = 0, and whether a negative L is possible. [3 marks]

A positive input less than one has a negative base-10 logarithm.

Answer: I = 0 gives no real logarithm; negative L is possible when $0 < I < I_0$.