Logarithmic functions - Practice test 4

Name: _____ Date: _____

10 questions | 54 marks | 70 minutes

Reasoning about domains, parameters, inverse graphs and an optimisation model.

A scientific calculator may be used.

1. Find real domains before simplifying. [5 marks]

(a) State the domain of $\ln(x+2)+\ln(4-x)$. [2 marks]

(b) Compare it with the domain of $\ln((x+2)(4-x))$. [3 marks]

2. Determine unknown logarithm bases. [5 marks]

(a) Find b if $\log_b 9 = 1/2$. [2 marks]

(b) Find c if $\log_c 16 = -2$. [3 marks]

3. For $u > 0$, define $H(u) = \ln u + \ln(1/u)$. [5 marks]

(a) Show that H(u) is constant and state its value. [2 marks]

(b) Differentiate the original two-term expression to confirm the result. [3 marks]

4. Solve $\ln(x^2 - 3x) = \ln 4$. [6 marks]

(a) Find the original domain. [3 marks]

(b) Find and check every real solution. [3 marks]

5. Consider $2^x + 2^{-x} = 5/2$. [6 marks]

(a) Use u = 2^x to find a quadratic equation in u. [3 marks]

(b) Find all real x. [3 marks]

6. Let $f(x) = \ln(3x)$, $x > 0$. [5 marks]

(a) Find its inverse and describe the relationship between the two graphs. [3 marks]

(b) Find the point on the inverse corresponding to (1/3, 0) on f. [2 marks]

7. Let $f(x) = a\ln x + bx$ for $x > 0$. Its tangent at x = 1 is horizontal and f(1) = 6. [6 marks]

(a) Determine a and b. [3 marks]

(b) Classify the stationary point at x = 1. [3 marks]

8. A mathematical cost model is $C(x) = 20 + x - 4\ln x$ for $x > 0$. [6 marks]

(a) Find the value of x minimising C and justify that it is a global minimum. [3 marks]

(b) Give the minimum exactly and explain why x = 0 cannot be tested as an endpoint. [3 marks]

9. Distance travelled in a supplied model is $D(t) = 12\ln(t+1)$ metres, $t \geq 0$ seconds. [5 marks]

(a) Find the time when D = 12 metres. [2 marks]

(b) If t + 1 doubles, show that the additional distance is constant and find it. [3 marks]

10. A student fits a formula without checking its useful domain. [5 marks]

(a) Find the mathematical domain of $M(t) = 50 + 10\ln(t-2)$. [2 marks]

(b) The data were observed only for $3 \leq t \leq 8$ days. Explain two separate cautions about predicting at t = 1 and t = 100. [3 marks]