Logarithmic functions - Practice test 5

Name: _____ Date: _____

10 questions | 57 marks | 70 minutes

Data interpretation, parameter fitting and checking claims about logarithmic models.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. Use a supplied table of powers. [5 marks]

Selected exact powers

n	3^n
-2	1/9
0	1
1	3
2	9
3	27

(a) Fill the logarithmic values for inputs 1/9, 1 and 27 in base 3. [2 marks]

(b) Estimate whether $\log_3 12$ lies between 1 and 2 or between 2 and 3, then calculate it to three decimal places. [3 marks]

2. Compare expressions and domains. [5 marks]

(a) For $x > 0$, simplify $\ln(8x) - \ln(2x)$. [2 marks]

(b) Is $\ln(x-2) + \ln(x+2) = \ln(x^2-4)$ valid for every x in the domain of the right side? Explain. [3 marks]

3. A coefficient is hidden inside an exponential model. [6 marks]

(a) Find k if $40e^{3k} = 100$. [3 marks]

(b) Using this exact k, solve $40e^{kt} = 250$. [3 marks]

4. The inverse function of f is $f^{-1}(x) = 4 + e^{x/2}$. [5 marks]

(a) Find f and its domain. [3 marks]

(b) Find the asymptote and the point at which f(x) = 0. [2 marks]

5. Let $f(x) = \ln(x^2 + 4x + 5)$. [6 marks]

(a) Explain why its domain is all real numbers. [2 marks]

(b) Find its stationary point and classify it. [4 marks]

6. A response is modelled by $R(t) = a + b\ln(t + 1)$ for $t \geq 0$ minutes. Observations are supplied. [7 marks]

Response observations

t (minutes)	R
0	5
e − 1	13

(a) Find a and b exactly. [3 marks]

(b) Find the time when R = 25 and the rate at that time. [4 marks]

7. Two graphs are $y = \ln x$ and $y = \ln x + \ln 5$. [5 marks]

(a) Rewrite the second as one logarithm and describe its horizontal scaling from the first. [3 marks]

(b) Explain how the same graph can also be described by a vertical translation. [2 marks]

8. A supplied scale is $B = \log_2 (N/N_0)$ for positive N and fixed positive N0. [5 marks]

(a) Find N/N0 when B = −3. [2 marks]

(b) Find the change in B if N is multiplied by 12. [3 marks]

9. A claimed derivative for $g(x) = \ln(4 - x^2)$ is $1/(4 - x^2)$. [6 marks]

(a) Give the correct derivative and domain. [3 marks]

(b) Use the correct derivative to determine where g is increasing. [3 marks]

10. A fitted score model is $S(t) = 25 + 15\ln(t + 1)$, based on observations for $0 \leq t \leq 6$ weeks. [7 marks]

(a) Predict the score at t = 6 and its instantaneous rate there. [3 marks]

(b) A researcher says the score will pass 100 eventually. Find the mathematical threshold and give a reason this may not be a reliable practical claim. [4 marks]