Initial-value problems

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. $F'(x) = 6x$ and $F(0) = 3$. Find F. [3 marks]

$F = 3x^2 + C$; at 0, $C = 3$.

Answer: $F = 3x^2 + 3$

2. $y' = 4x^3$ and $y(1) = 7$. Find y. [3 marks]

$7 = 1 + C$ gives $C = 6$.

Answer: $y = x^4 + 6$

3. $f'(x) = \cos x$ and $f(\pi/2) = 4$. Find f. [3 marks]

Integrate to $\sin x + C$; $4 = 1 + C$.

Answer: $f(x) = \sin x + 3$

4. $g'(x) = 2/x$ for $x > 0$ and $g(1) = 5$. Find g. [3 marks]

$\ln 1 = 0$, so $C = 5$.

Answer: $g(x) = 2\ln x + 5$

5. A particle has acceleration $a(t) = 6t$ m/s² and initial velocity 2 m/s. Find its velocity at $t = 3$ s. [3 marks]

$v(t) = 3t^2 + C$.

Use $v(0) = 2$ to obtain $C = 2$, then $v(3) = 29$.

Answer: 29 m/s

6. A quantity satisfies $N'(t) = 3e^t$ and $N(0) = 8$. Find $N(\ln 2)$. [3 marks]

$N = 3e^t + 5$.

$N(\ln 2) = 3(2) + 5 = 11$.

Answer: 11