Signed area versus geometric area

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. Find the area between $y = x - 2$ and the axis on $[0, 4]$. [3 marks]

Split at 2.

Two triangles each have area 2.

Answer: 4

2. Find $\int_{-2}^{2} x\,dx$ and the corresponding geometric area. [3 marks]

The signed contributions are -2 and 2.

Their absolute values sum to 4.

Answer: 0 and 4

3. Find the area under $y = 4 - x^2$ on $[-2, 2]$. [3 marks]

The function is nonnegative, so integrate it directly.

Answer: 32/3

4. Find the area between $y = x(x - 2)$ and the axis on $[0, 3]$. [3 marks]

An antiderivative is $x^3/3 - x^2$.

Contributions on $[0, 2]$ and $[2, 3]$ are $-4/3$ and $4/3$.

Add their magnitudes.

Answer: 8/3

5. Find the area between $y = \sin x$ and the axis on $[0, 2\pi]$. [3 marks]

On $[0, \pi]$ the area is 2; on $[\pi, 2\pi]$ the signed integral is -2.

Add 2 and 2.

Answer: 4

6. A student says an integral of 0 proves that an area is 0. Give a counterexample. [3 marks]

The integral is 0 but the two triangular regions have total area 1.

Answer: $f(x) = x$ on $[-1, 1]$.