Contextual integral models

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. Find the average value of $f(x) = 3x^2$ on $[0, 2]$. [3 marks]

The integral is 8; divide by interval length 2.

Answer: 4

2. $C'(q) = 2q + 5$ dollars/item and $C(0) = 40$. Find $C(6)$. [3 marks]

Change $= [q^2 + 5q]_0^6 = 36 + 30 = 66$.

Add 40.

Answer: 106 dollars

3. Water flows at $12e^{-t}$ L/min for $0 \le t \le \ln 2$. Find the supplied volume. [3 marks]

$[-12e^{-t}]_0^{\ln 2} = -6 + 12 = 6$.

Answer: 6 L

4. A force $F(x) = 4x$ newtons moves an object from $x = 1$ to $x = 3$ metres. Find $\int_1^3 F(x)\,dx$ and its unit. [3 marks]

$[2x^2]_1^3 = 18 - 2 = 16$ N·m.

One newton-metre is one joule.

Answer: 16 joules

5. A rate model $r(t) = 10 - 2t$ items/hour is used for production until $t = 8$ hours. Explain a limitation. [3 marks]

The zero is $t = 5$; a revised model or valid interval is needed beyond that time.

Answer: The model predicts negative production after 5 hours.

6. The average value of $f(x) = kx$ on $[0, 4]$ is 6. Find k. [3 marks]

$\frac{1}{4}\int_0^4 kx\,dx = 2k$.

Solve $2k = 6$.

Answer: $k = 3$