Integration - Practice test 1

10 questions | 41 marks | 70 minutes

A balanced first paper: rules, interpretation, area and motion.

Equivalent correct methods earn the corresponding marks.

1. Antiderivatives. [4 marks]

(a) Find $\int (8x^3 - 6x + 5)\,dx$. [2 marks]

Integrate each term.

Answer: $2x^4 - 3x^2 + 5x + C$

(b) Differentiate your answer to check. [2 marks]

Differentiate term by term.

Answer: $8x^3 - 6x + 5$

2. Linear inputs. [4 marks]

(a) Find $\int 3e^{2x}\,dx$. [2 marks]

Divide by 2.

Answer: $\frac{3}{2}e^{2x} + C$

(b) Find $\int 4\cos(2x)\,dx$. [2 marks]

Divide by 2.

Answer: $2\sin(2x) + C$

3. A reciprocal expression. [4 marks]

(a) Find $\int \frac{6}{3x+2}\,dx$. [2 marks]

Divide 6 by 3.

Answer: $2\ln|3x + 2| + C$

(b) Evaluate $\int_0^1 \frac{6}{3x+2}\,dx$. [2 marks]

$2\ln 5 - 2\ln 2 = 2\ln(5/2)$.

Answer: $2\ln(5/2)$

4. $f'(x) = 3x^2 + 2$ and $f(1) = 6$. [4 marks]

(a) Find $f(x)$. [2 marks]

$6 = 1 + 2 + C$ gives $C = 3$.

Answer: $x^3 + 2x + 3$

(b) Find $f(2)$. [2 marks]

$8 + 4 + 3 = 15$.

Answer: 15

5. Definite-integral structure. [4 marks]

(a) Evaluate $\int_1^3 (2x + 1)\,dx$. [2 marks]

$[x^2 + x]_1^3 = 12 - 2 = 10$.

Answer: 10

(b) Hence find $\int_3^1 (6x + 3)\,dx$. [2 marks]

Reverse the bounds and multiply by 3.

Answer: -30

6. The speed samples shown are used to estimate distance. [4 marks]

Speed recorded at equally spaced times.

Time (s)	0	1	2	3
Speed (m/s)	0	2	3	6

(a) Apply the trapezium rule. [2 marks]

Widths 1; strip areas 1,2.5,4.5.

Answer: 8 m

(b) Explain one assumption behind this estimate. [2 marks]

The true intermediate speeds are unknown.

Answer: A straight-line speed change is used between samples.

7. $A(x) = \int_0^{2x} (t^2 + 1)\,dt$. [4 marks]

(a) Find $A'(x)$ without evaluating the integral. [2 marks]

$[(2x)^2 + 1]2 = 8x^2 + 2$.

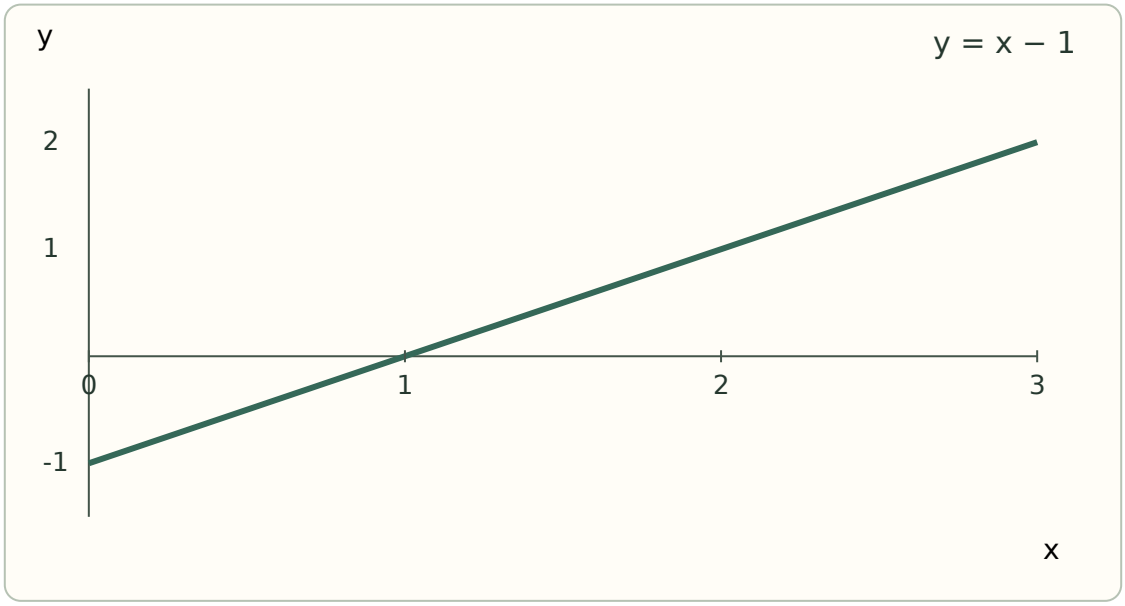
Answer: $8x^2 + 2$

(b) Find $A(0)$ and interpret it. [2 marks]

The integration interval has zero width.

Answer: 0

8. The graph shows $y = x - 1$ on $[0, 3]$. [4 marks]



(a) Find the signed integral. [2 marks]

$[x^2/2 - x]_0^3 = 3/2$.

Answer: 3/2

(b) Find the geometric area. [2 marks]

Split at 1; add areas 1/2 and 2.

Answer: 5/2

9. The curves $y = x^2$ and $y = 2x$ enclose a region. [4 marks]

(a) Find the intersection points. [2 marks]

$x(x - 2) = 0$.

Answer: $(0, 0)$ and $(2, 4)$

(b) Find the enclosed area. [2 marks]

Integrate $2x - x^2$ from 0 to 2.

Answer: 4/3

10. A particle has $v(t) = t - 2$ m/s on $0 \le t \le 5$ and initial position 4 m. [5 marks]

(a) Find its final position. [2 marks]

Displacement 5/2 m; add 4 m.

Answer: 13/2 m

(b) Find its total distance. [3 marks]

Split at $t = 2$:2+9/2=13/2 m.

Answer: 13/2 m