Integration - Practice test 4

10 questions | 44 marks | 70 minutes

A varied fourth paper emphasising trigonometric, exponential and logarithmic accumulation.

Equivalent correct methods earn the corresponding marks.

1. Reverse differentiation. [4 marks]

(a) Find $\int (4x^3 - 2e^{-x})\,dx$. [2 marks]

The negative exponential input changes the sign.

Answer: $x^4 + 2e^{-x} + C$

(b) Find $\int 5\cos(5x)\,dx$. [2 marks]

The input factor cancels 5.

Answer: $\sin(5x) + C$

2. Logarithmic forms. [4 marks]

(a) Find $\int \frac{3}{2-x}\,dx$. [2 marks]

Input derivative -1.

Answer: $-3\ln|2-x| + C$

(b) Evaluate $\int_0^1 \frac{3}{2-x}\,dx$. [2 marks]

$[-3\ln(2-x)]_0^1 = 3\ln 2$.

Answer: $3\ln 2$

3. A curve satisfies $y' = 2\sin x$ and $y(0) = 3$. [4 marks]

(a) Find y. [2 marks]

$3 = -2 + C$.

Answer: $y = -2\cos x + 5$

(b) Find its net change from 0 to π. [2 marks]

$y(\pi) - y(0) = 7 - 3 = 4$.

Answer: 4

4. A continuous function has $\int_0^3 f = 4$. [4 marks]

(a) Find $\int_0^3 (2f(x) + 1)\,dx$. [2 marks]

$2(4) + (3 - 0) = 11$.

Answer: 11

(b) Find $\int_3^0 (2f(x) + 1)\,dx$. [2 marks]

Reverse the limits.

Answer: -11

5. $A(x) = \int_0^{x+2} e^t\,dt$. [4 marks]

(a) Find A'. [2 marks]

The endpoint derivative is 1.

Answer: e^{x+2}

(b) Evaluate $A(0)$. [2 marks]

$[e^t]_0^2 = e^2 - 1$.

Answer: $e^2 - 1$

6. For $f(x) = x^2$ on $[0, 2]$. [6 marks]

(a) Calculate the exact integral. [2 marks]

$[x^3/3]_0^2 = 8/3$.

Answer: 8/3

(b) The two-strip trapezium estimate is 3. Find the absolute error. [2 marks]

$3 - 8/3 = 1/3$.

Answer: 1/3

(c) Explain the direction of this error. [2 marks]

The straight chords lie above the curve.

Answer: Overestimate because the curve is convex.

7. Consider $y = \sin x$ on $[0, 2\pi]$. [5 marks]

(a) Find the signed integral. [2 marks]

$[-\cos x]_0^{2\pi} = 0$.

Answer: 0

(b) Find geometric area. [3 marks]

Split at π and add magnitudes 2+2.

Answer: 4

8. Find the bounded region between $y = x$ and $y = x^2$. [5 marks]

(a) Identify which curve is higher between intersections. [2 marks]

Test $x = 1/2$.

Answer: $y = x$ on $0 < x < 1$

(b) Find area. [3 marks]

Integrate $x - x^2$ from 0 to 1.

Answer: 1/6

9. A tank receives $12e^{-t}$ L/min for $0 \le t \le \ln 2$. [4 marks]

(a) Find the supplied volume. [2 marks]

$[-12e^{-t}]_0^{\ln 2} = 6$.

Answer: 6 L

(b) Find the average flow rate. [2 marks]

Divide total volume by interval duration.

Answer: $6/\ln 2$ L/min

10. A particle has $v(t) = \cos t$ m/s on $[0, \pi]$ and initial position 2 m. [4 marks]

(a) Find its final position. [2 marks]

The signed velocity integral is 0.

Answer: 2 m

(b) Find its distance. [2 marks]

Direction changes at $\pi/2$; magnitudes 1+1.

Answer: 2 m