Expected value

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. X takes values 1, 2 and 4 with probabilities 0.2, 0.5 and 0.3. Find E(X). [2 marks]

E(X) = 1(0.2) + 2(0.5) + 4(0.3).

The sum is 0.2 + 1 + 1.2 = 2.4.

Answer: 2.4.

2. A fair die is rolled and X is its score. Find E(X). [2 marks]

Each score has probability 1/6.

E(X) = (1 + 2 + 3 + 4 + 5 + 6)/6 = 21/6.

Answer: 3.5.

3. A ticket pays 20 dollars with probability 0.1 and 0 dollars otherwise. It costs 3 dollars. Find expected net gain. [2 marks]

Expected prize is 20(0.1) = 2 dollars.

Net expected gain = 2 − 3 = −1 dollar.

Answer: −1 dollar.

4. X is 0 or 5. If E(X) = 2, find P(X = 5). [2 marks]

Let p = P(X = 5), so E(X) = 0(1 − p) + 5p.

Solve 5p = 2 to obtain p = 0.4.

Answer: 0.4.

5. A machine makes 0, 1 or 2 faulty items per batch with probabilities 0.7, 0.2 and 0.1. Find the expected number over 50 batches. [3 marks]

Expected faults per batch = 0.2 + 2(0.1) = 0.4.

By linearity of expectation, the expected total is 50(0.4) = 20.

Answer: 20 faulty items.

6. A forecast says E(X) = 2.4 calls per minute. Explain why 2.4 need not be a possible call count. [2 marks]

Each observed count is an integer.

A probability-weighted mean can be non-integer without contradicting the model.

Answer: It is a long-run average, not an individual count.