Variance and standard deviation

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. X is 0 or 2 with equal probabilities. Find its mean and variance. [2 marks]

$E(X) = 1$; $E(X^2) = 2$.

$\text{Var}(X) = 2 - 1^2 = 1$.

Answer: Mean 1; variance 1.

2. $E(X) = 3$ and $E(X^2) = 14$. Find variance and standard deviation. [2 marks]

$\text{Var}(X) = 14 - 9 = 5$.

Standard deviation is the non-negative square root.

Answer: 5; $\sqrt{5} \approx 2.2361$.

3. X is -1 or 1 with equal probabilities. Find its mean and standard deviation. [2 marks]

$E(X) = (-1 + 1)/2 = 0$.

X^2 is always 1, so variance is 1 and standard deviation is 1.

Answer: Mean 0; standard deviation 1.

4. A payout in dollars has variance 9. State its standard deviation and units. [2 marks]

Standard deviation = $\sqrt{9} = 3$.

Its units are dollars, unlike the squared-dollar variance.

Answer: 3 dollars.

5. X has values 1, 2 and 3 with probabilities 1/4, 1/2 and 1/4. Find $E(X^2)$ and Var(X). [3 marks]

$E(X) = 1/4 + 1 + 3/4 = 2$.

$E(X^2) = 1/4 + 2 + 9/4 = 9/2$.

$\text{Var}(X) = 9/2 - 4 = 1/2$.

Answer: $E(X^2) = 9/2;\ \text{Var}(X) = 1/2$.

6. A student obtains Var(X) = −0.8 from a valid exact probability table. Explain why this cannot be accepted. [2 marks]

Every squared deviation is non-negative and probabilities are non-negative.

Their weighted sum cannot be below zero; recheck the calculation.

Answer: Variance cannot be negative.