Effects of scale/shift

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. E(X) = 5 and Var(X) = 4. Find E(2X + 3) and Var(2X + 3). [2 marks]

E(2X + 3) = 2(5) + 3.

Var(2X + 3) = 2^2(4).

Answer: 13 and 16.

2. E(X) = 8 and SD(X) = 3. Find the mean and SD of Y = 10 − X. [2 marks]

E(Y) = 10 − 8.

The scale magnitude is 1, so the standard deviation stays 3.

Answer: Mean 2; SD 3.

3. A random length X in metres has mean 1.2 and variance 0.04. Convert both to centimetre-based quantities. [3 marks]

Y = 100X.

E(Y) = 100(1.2) = 120; Var(Y) = 100^2(0.04) = 400.

Answer: Mean 120 cm; variance 400 cm^2.

4. A score X has mean 50 and SD 10. Find mean and variance of $Z = (X - 50)/10$. [2 marks]

$E(Z) = (50 - 50)/10 = 0$.

$Var(Z) = 100/100 = 1$.

Answer: Mean 0; variance 1.

5. X is −2 or 2 with equal probabilities. Find $E(X^2)$. Explain why squaring E(X) gives the wrong answer. [3 marks]

X^2 always equals 4, so its mean is 4.

$E(X) = 0$, but a nonlinear transformation cannot be moved through expectation.

Answer: $E(X^2) = 4$; $[E(X)]^2 = 0$.

6. X is 1, 2 or 4 with probabilities 0.2, 0.5, 0.3. For $Y = 6 - X$, find $P(Y \geq 4)$. [2 marks]

$6 - X \geq 4$ gives $X \leq 2$.

Add probabilities at X = 1 and X = 2: 0.2 + 0.5.

Answer: 0.7.