Sums of independent variables

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. Independent X and Y have means 4, 7 and variances 1, 9. Find the mean and variance of X + Y. [2 marks]

E(X + Y) = 4 + 7.

Var(X + Y) = 1 + 9 by independence.

Answer: 11 and 10.

2. Using the same X and Y, find the mean and variance of 3X − 2Y + 5. [3 marks]

Mean = 3(4) − 2(7) + 5 = 3.

Variance = 9(1) + 4(9) = 45.

Answer: Mean 3; variance 45.

3. Five independent copies of X each have mean 2 and variance 3. Find the mean and variance of their sum. [2 marks]

Add five means: 5(2).

Add five independent variances: 5(3).

Answer: Mean 10; variance 15.

4. For those five independent copies, find the mean and variance of their average. [3 marks]

The average is one fifth of the sum.

Its mean is 10/5 = 2; variance is 15/25 = 3/5.

Answer: 2; 3/5.

5. Independent fair dice are rolled. Let X and Y be their scores. Find P(X + Y = 5). [2 marks]

The ordered pairs are (1,4), (2,3), (3,2), (4,1).

Each has probability 1/36, so the required probability is 4/36.

Answer: 1/9.

6. A supplier gives E(X) = 8, E(Y) = 3, Var(X) = 4 and Var(Y) = 2, but does not state independence. Which of E(X + Y) = 11 and Var(X + Y) = 6 is justified? [3 marks]

Expectation is additive without independence.

The variance calculation also needs independence or suitable dependence information.

Answer: Only the mean statement is justified.