Bernoulli trials

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. A coin lands heads with probability 0.55. Define a Bernoulli indicator for tails and give p. [2 marks]

The chosen success event is tails.

Its probability is $1 - 0.55$.

Answer: B = 1 for tails, 0 for heads; p = 0.45.

2. B is Bernoulli with p = 0.4. Find E(B), Var(B) and SD(B). [3 marks]

Use E(B) = p.

Use Var(B) = p(1 − p) and take its square root.

Answer: $0.4;\ 0.24;\ \sqrt{0.24} \approx 0.4899$.

3. Let B be Bernoulli with p = 0.1. A bonus is Y = 20B dollars. Find its mean and variance. [3 marks]

E(Y) = 20(0.1) = 2.

Var(Y) = $20^2(0.1)(0.9) = 36$.

Answer: Mean 2 dollars; variance 36 dollars^2.

4. If B is Bernoulli with p = 1, describe B and find its variance. [2 marks]

Success is certain.

The formula p(1 − p) gives 1 × 0 = 0.

Answer: B is always 1; variance 0.

5. A variable equals 2 or 7. Explain how a Bernoulli variable B can represent it. [2 marks]

If B = 0 the expression is 2.

If B = 1 it is 7; p is the probability of the outcome 7.

Answer: X = 2 + 5B, with B = 1 for the outcome 7.

6. Show that a Bernoulli variance cannot exceed 1/4. [3 marks]

Complete the square in $p - p^2$.

A square is non-negative, with equality in the bound when p = 1/2.

Answer: $p(1-p) = 1/4 - (p - 1/2)^2 \leq 1/4$.