Discrete distributions - Practice test 1

10 questions | 58 marks | 70 minutes

Build confidence with tables, cumulative probabilities, summary measures and an ordinary binomial model.

Equivalent correct methods earn the corresponding marks.

1. X has the probability distribution shown. [6 marks]

Distribution of X

x	0	1	2	4
P(X = x)	0.2	0.3	k	0.1

(a) Find k. [2 marks]

Probabilities sum to 1: $k = 1 - 0.2 - 0.3 - 0.1$.

Answer: $k = 0.4$.

(b) Calculate E(X). [2 marks]

$E(X) = 0(0.2) + 1(0.3) + 2(0.4) + 4(0.1) = 1.5$.

Answer: 1.5.

(c) Find $P(X \geq 2)$. [2 marks]

Add probabilities for 2 and 4: 0.4 + 0.1.

Answer: 0.5.

2. X is integer-valued and has F(0) = 0.1, F(1) = 0.4, F(2) = 0.75 and F(3) = 1. It takes no other values. [6 marks]

(a) Find P(X = 2). [2 marks]

The jump at 2 is F(2) − F(1) = 0.75 − 0.4.

Answer: 0.35.

(b) Find P(1 ≤ X ≤ 2). [2 marks]

Subtract F(0) from F(2): 0.75 − 0.1.

Answer: 0.65.

(c) Find P(X > 1). [2 marks]

Use 1 − F(1) = 1 − 0.4.

Answer: 0.6.

3. A game pays 0 dollars, 5 dollars or 20 dollars with probabilities 0.6, 0.3 and 0.1. The entry fee is 4 dollars. [6 marks]

(a) Find the expected gross prize. [2 marks]

0(0.6) + 5(0.3) + 20(0.1) = 3.5.

Answer: 3.50 dollars.

(b) Find the fair fee and expected net gain at the actual fee. [2 marks]

A fair fee equals the gross expected prize.

Subtract the actual fee: 3.50 − 4.00 = −0.50.

Answer: Fair fee 3.50 dollars; expected net gain −0.50 dollars.

(c) Find expected total net gain over 200 plays. [2 marks]

Add expected gains: 200(−0.50) = −100.

Answer: −100 dollars.

4. X takes values −1, 1 and 3 with probabilities 1/4, 1/2 and 1/4. [5 marks]

(a) Find E(X) and E(X²). [2 marks]

E(X) = −1/4 + 1/2 + 3/4 = 1.

E(X²) = 1/4 + 1/2 + 9/4 = 3.

Answer: 1 and 3.

(b) Find Var(X) and SD(X). [2 marks]

Var(X) = 3 − 1² = 2.

SD is the positive square root of variance.

Answer: 2; $\sqrt{2} \approx 1.4142$.

(c) Explain why the variance is not expressed in the same units as X. [1 mark]

If X has units u, the variance has units u².

Answer: The deviations are squared.

5. A score X has mean 8 and variance 2. A conversion uses Y = 3X − 5. [6 marks]

(a) Find E(Y). [2 marks]

3(8) − 5 = 19.

Answer: 19.

(b) Find Var(Y) and SD(Y). [2 marks]

Variance multiplies by 3^2: 9(2) = 18.

The fixed shift does not affect spread.

Answer: 18; $\sqrt{18} \approx 4.2426$.

(c) If the subtraction of 5 is removed, what happens to the mean and variance? [2 marks]

A fixed shift changes every outcome by the same amount.

Answer: The mean increases by 5 to 24; variance remains 18.

6. Independent X and Y have means 4 and 3 and variances 5 and 2. [5 marks]

(a) Find the mean and variance of 2X + Y. [2 marks]

E(2X + Y) = 2(4) + 3.

Var(2X + Y) = 4(5) + 2 by independence.

Answer: Mean 11; variance 22.

(b) Find the mean and variance of X − Y. [2 marks]

E(X − Y) = 4 − 3.

Var(X − Y) = 5 + 2.

Answer: Mean 1; variance 7.

(c) Which calculation in part (b) still works if independence is not known? [1 mark]

Linearity of expectation does not need independence; adding variances does here.

Answer: The mean calculation.

7. A check passes with probability 0.8. Let B = 1 for a failure and B = 0 for a pass. [6 marks]

(a) State the distribution and mean of B. [2 marks]

Success for this indicator is failure of the check.

Answer: P(B = 1) = 0.2, P(B = 0) = 0.8; mean 0.2.

(b) Find Var(B). [2 marks]

0.2(0.8) = 0.16.

Answer: 0.16.

(c) Find the mean and variance of the score 4B − 1. [2 marks]

Mean = 4(0.2) − 1.

Variance = 4^2(0.16).

Answer: Mean −0.2; variance 2.56.

8. A biased coin has P(heads) = 0.3 and is tossed independently five times. X counts tails. [5 marks]

(a) State a binomial model for X and justify its parameters. [3 marks]

There are five fixed independent trials, tails/no tails categories and common tails probability 0.7.

Answer: $X \sim \text{Bin}(5, 0.7)$.

(b) A different process changes the coin bias after each toss. Explain which binomial condition may fail. [2 marks]

Changing the bias can make the tail probability different on successive trials.

Answer: The common success probability condition.

9. $X \sim \text{Bin}(8, 0.3)$. Give probabilities to four decimal places. [6 marks]

(a) Find P(X = 2). [2 marks]

$\binom{8}{2}(0.3)^2(0.7)^6 = 0.29647548.$

Answer: 0.2965

(b) Find P(X ≥ 2). [2 marks]

Use $1 - P(X = 0) - P(X = 1)$.

$\sum_{k=2}^{8} \binom{8}{k}(0.3)^k(0.7)^{8-k} = 0.7447.$

Answer: 0.7447

(c) State E(X) and explain why it need not be a possible outcome. [2 marks]

E(X) = 8(0.3) = 2.4; a single count is still an integer.

Answer: 2.4; it is a long-run mean.

10. A binomial variable has exact mean 12 and exact variance 9. [7 marks]

(a) Find n and p. [3 marks]

1 − p = 9/12 = 0.75, hence p = 0.25.

n = 12/0.25 = 48.

Answer: n = 48, p = 0.25.

(b) Find the probability of zero successes, using scientific notation if useful. [2 marks]

$P(X = 0) = 0.75^{48}$.

Answer: 1.00679e-06

(c) Does the mean force each set of 48 trials to contain 12 successes? Explain. [2 marks]

The mean averages counts over many repeated sets; any count from 0 to 48 remains possible for 0 < p < 1.

Answer: No.