Discrete distributions - Practice test 1

Name: _____ Date: _____

10 questions | 58 marks | 70 minutes

Build confidence with tables, cumulative probabilities, summary measures and an ordinary binomial model.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. X has the probability distribution shown. [6 marks]

Distribution of X

x	0	1	2	4
P(X = x)	0.2	0.3	k	0.1

(a) Find k. [2 marks]

(b) Calculate E(X). [2 marks]

(c) Find P(X ≥ 2). [2 marks]

2. X is integer-valued and has F(0) = 0.1, F(1) = 0.4, F(2) = 0.75 and F(3) = 1. It takes no other values. [6 marks]

(a) Find P(X = 2). [2 marks]

(b) Find P(1 ≤ X ≤ 2). [2 marks]

(c) Find P(X > 1). [2 marks]

3. A game pays 0 dollars, 5 dollars or 20 dollars with probabilities 0.6, 0.3 and 0.1. The entry fee is 4 dollars. [6 marks]

(a) Find the expected gross prize. [2 marks]

(b) Find the fair fee and expected net gain at the actual fee. [2 marks]

(c) Find expected total net gain over 200 plays. [2 marks]

4. X takes values -1, 1 and 3 with probabilities 1/4, 1/2 and 1/4. [5 marks]

(a) Find E(X) and $E(X^2)$. [2 marks]

(b) Find Var(X) and SD(X). [2 marks]

(c) Explain why the variance is not expressed in the same units as X. [1 mark]

5. A score X has mean 8 and variance 2. A conversion uses $Y = 3X - 5$. [6 marks]

(a) Find E(Y). [2 marks]

(b) Find Var(Y) and SD(Y). [2 marks]

(c) If the subtraction of 5 is removed, what happens to the mean and variance? [2 marks]

6. Independent X and Y have means 4 and 3 and variances 5 and 2. [5 marks]

(a) Find the mean and variance of $2X + Y$. [2 marks]

(b) Find the mean and variance of $X - Y$. [2 marks]

(c) Which calculation in part (b) still works if independence is not known? [1 mark]

7. A check passes with probability 0.8. Let B = 1 for a failure and B = 0 for a pass. [6 marks]

(a) State the distribution and mean of B. [2 marks]

(b) Find Var(B). [2 marks]

(c) Find the mean and variance of the score $4B - 1$. [2 marks]

8. A biased coin has P(heads) = 0.3 and is tossed independently five times. X counts tails. [5 marks]

(a) State a binomial model for X and justify its parameters. [3 marks]

(b) A different process changes the coin bias after each toss. Explain which binomial condition may fail. [2 marks]

9. $X \sim \text{Bin}(8, 0.3)$. Give probabilities to four decimal places. [6 marks]

(a) Find P(X = 2). [2 marks]

(b) Find P(X ≥ 2). [2 marks]

(c) State E(X) and explain why it need not be a possible outcome. [2 marks]

10. A binomial variable has exact mean 12 and exact variance 9. [7 marks]

(a) Find n and p. [3 marks]

(b) Find the probability of zero successes, using scientific notation if useful. [2 marks]

(c) Does the mean force each set of 48 trials to contain 12 successes? Explain. [2 marks]