Discrete distributions - Practice test 2

Name: _____ Date: _____

10 questions | 64 marks | 70 minutes

Compare models, recover a distribution from cumulative information and examine the role of independence.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. A bag contains two red and one blue counter. Two counters are selected fairly without replacement. X is the red count. [8 marks]

(a) Construct the distribution of X. [3 marks]

(b) Calculate E(X). [2 marks]

(c) Explain why a Bin(2, 2/3) model has the correct mean but is not the correct distribution. [3 marks]

2. X takes only -2, 0 and 5. Its cumulative probabilities at these values are 0.2, 0.7 and 1. [6 marks]

(a) Find $P(X = 0)$. [2 marks]

(b) Find $P(-1 < X < 5)$. [2 marks]

(c) Find $F(3)$. [2 marks]

3. X takes values 0, 2 and 8 with probabilities a, 0.5 and 0.5 − a. Its expected value is 3. [6 marks]

(a) Find a. [3 marks]

(b) Find Var(X). [2 marks]

(c) Check that all probabilities are valid. [1 mark]

4. Option A pays 0 or 2 with equal probabilities. Option B always pays 1. [6 marks]

(a) Show that their means are equal. [2 marks]

(b) Compare their variances. [2 marks]

(c) Which option avoids the possibility of receiving zero? Explain. [2 marks]

5. X takes 0, 1 and 3 with probabilities 0.1, 0.2 and 0.7. Let $Y = 10 - 2X$. [7 marks]

(a) Give the values and probabilities of Y. [2 marks]

(b) Find $P(Y \geq 8)$. [2 marks]

(c) Find the mean and variance of Y. [3 marks]

6. Independent success indicators B and C have success probabilities 0.2 and 0.6. Set S = B + C. [7 marks]

(a) Find the distribution of S. [3 marks]

(b) Find its mean and variance. [2 marks]

(c) Why is S not Bin(2, 0.4)? [2 marks]

7. A fair coin is tossed once. B = 1 for heads and C is defined to equal B. Let S = B + C. [6 marks]

(a) Construct the distribution of S and find Var(S). [2 marks]

(b) Why is adding the two Bernoulli variances invalid here? [2 marks]

(c) Does E(S) = E(B) + E(C) still hold? Explain. [2 marks]

8. A group has 3 marked and 17 unmarked items. Two are sampled without replacement. [6 marks]

(a) Find the exact probability that both are marked. [2 marks]

(b) Find the probability predicted by Bin(2, 0.15) for two marked items. [2 marks]

(c) Explain the source of the difference. [2 marks]

9. $X \sim \text{Bin}(6, 0.4)$. Give probabilities to four decimal places. [6 marks]

(a) Find $P(1 \leq X \leq 3)$. [2 marks]

(b) Find $P(X > 3)$. [2 marks]

(c) Among 80 comparable groups, how many are expected to have more than three successes? [2 marks]

10. A device works if at least 3 of its 4 independent components work. Each component works with probability p. [6 marks]

(a) Find the device reliability for p = 0.9. [2 marks]

(b) Find the reliability for p = 0.95. [2 marks]

(c) Which component choice meets a device target of at least 0.98? [2 marks]