Discrete distributions - Practice test 4

Name: _____ Date: _____

10 questions | 64 marks | 70 minutes

Explain distribution identities, distinguish dependent indicators and solve a minimum-trial problem.

Use a graphics calculator or appropriate digital tool for labelled tasks.

1. X takes values 0, 1, 2 and 3, with $P(X = x) = k(x + 1)$. [6 marks]

(a) Find k and P(X ≥ 2). [2 marks]

(b) Find E(X) and Var(X). [2 marks]

(c) Find F(1.5). [2 marks]

2. X takes values 0, 1, 2 and 3. Its cumulative values are F(0) = 0.15, F(1) = a, F(2) = 0.8 and F(3) = 1. Also P(X = 1) = 0.35. [6 marks]

(a) Find a. [2 marks]

(b) Find P(X = 2). [2 marks]

(c) Find P(X ≥ 2) and explain the endpoint. [2 marks]

3. X takes −2, 0 and 2 with probabilities 1/4, 1/2 and 1/4. Let $Y = X^2$. [7 marks]

(a) Find E(X), $E(X^2)$ and Var(X). [2 marks]

(b) Construct the distribution of Y and find its variance. [3 marks]

(c) Explain why $E(X^2)$ is not $[E(X)]^2$ here. [2 marks]

4. D is the score on a fair six-sided die. [6 marks]

(a) Calculate $E(D^2)$. [2 marks]

(b) Use E(D) = 3.5 to find Var(D). [2 marks]

(c) For two independent dice, find the variance of their total. [2 marks]

5. X has mean $\mu = 30$ and standard deviation $\sigma = 6$. Define $Z = (X - 30)/6$. [6 marks]

(a) Find E(Z) and Var(Z). [2 marks]

(b) Express the event Z > 2 as an event involving X. [2 marks]

(c) Can P(Z > 2) be obtained from the supplied mean and SD alone? [2 marks]

6. Three independent indicators have success probabilities 0.1, 0.4 and 0.8. T is their total. [6 marks]

(a) Find its mean and variance. [2 marks]

(b) Find P(T = 0) and P(T = 3). [2 marks]

(c) Explain why matching the mean with Bin(3, 1.3/3) does not establish that model. [2 marks]

7. A fair coin defines $B = 1$ for heads. Let $C = 1 - B$ and $T = B + C$. [6 marks]

(a) Find $E(T)$ and $Var(T)$ directly. [2 marks]

(b) Calculate what an incorrect independence assumption would give for $Var(T)$. [2 marks]

(c) Explain the dependence. [2 marks]

8. $X \sim \text{Bin}(2, p)$, where $0 < p < 1$, and $P(X = 2) = 9P(X = 0)$. [7 marks]

(a) Form and solve an equation for p. [3 marks]

(b) Find P(X = 1). [2 marks]

(c) Find the mean and variance. [2 marks]

9. Independent trials have success probability 0.2. A planner wants the probability of at least one success to be at least 0.95. [7 marks]

(a) Write the required inequality in n. [2 marks]

(b) Find the smallest integer n. [3 marks]

(c) Verify the boundary using $n - 1$ and n. [2 marks]

10. Game A charges 2 dollars and pays 10 dollars with probability 0.2, otherwise 0 dollars. Game B charges 2 dollars and always returns 2 dollars. A player can choose either. [7 marks]

(a) Compare expected net gains. [2 marks]

(b) Compare net-gain variances. [2 marks]

(c) For ten independent plays of A, find the probability of a strictly positive total net gain. [3 marks]