Interval probabilities by integration

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. For density $2x$ on $[0,1]$, find $P(0.3 < X < 0.9)$. [2 marks]

$0.9^2 - 0.3^2 = 0.72$.

Answer: 0.72.

2. For density $3x^2$ on $[0,1]$, find $P(X < 0.2)$. [2 marks]

$[x^3]_0^{0.2} = 0.2^3$.

Answer: 0.008.

3. A waiting time is uniform on $[0,12]$ minutes. Find the chance of waiting between 3 and 8 minutes. [2 marks]

The density is 1/12 and the event width is 5.

Answer: 5/12.

4. For density $6x(1-x)$ on $[0,1]$, find $P(1/4 < X < 3/4)$. [3 marks]

Use $3x^2 - 2x^3$ as an antiderivative.

Its endpoint difference is $27/32 - 5/32 = 11/16$.

Answer: 11/16.

5. For density $2x$ on $[0,1]$, find $P(X > 0.6 \mid X > 0.3)$. [3 marks]

The numerator is $1 - 0.6^2 = 0.64$.

The denominator is $1 - 0.3^2 = 0.91$; divide.

Answer: 64/91.

6. Explain why including either endpoint of a continuous interval does not change its probability. [2 marks]

A finite collection of singleton values adds no area.

Answer: Each endpoint has probability zero.