Cumulative distribution functions

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. For CDF x^2 on [0,1], find $P(0.4 < X < 0.8)$. [2 marks]

$0.8^2 - 0.4^2 = 0.48$.

Answer: 0.48.

2. Give the CDF of density $3x^2$ on [0,1] over the whole real line. [3 marks]

Integrate $3t^2$ from 0 to x inside the support; use outer cumulative limits.

Answer: 0 below 0; x^3 on [0,1]; 1 above 1.

3. Find the 90th percentile for CDF x^2 on [0,1]. [2 marks]

Solve $q^2 = 0.9$ in the support.

Answer: $\sqrt{0.9} \approx 0.9487$.

4. Can a CDF decrease from 0.6 to 0.5 as x increases? Explain. [2 marks]

An increasing upper bound cannot remove already accumulated probability.

Answer: No.

5. X is uniform on [4,10]. Find F(2) and F(12). [2 marks]

2 is below the support; 12 is above it.

Answer: 0 and 1.

6. For CDF $1 - (1 - x)^2$ on [0,1], find the density and median. [3 marks]

Differentiate the CDF.

Solve $(1 - m)^2 = 1/2$ and retain the root in [0,1].

Answer: $f(x) = 2(1 - x)$; median $1 - 1/\sqrt{2}$.