Independent sums and sample means

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. Independent $X \sim N(10, 4)$, $Y \sim N(5, 9)$. Find the distribution of X+Y. [2 marks]

Means add and independent variances add.

Answer: $N(15, 13)$.

2. A normal population has mean 80 and SD 12. Find the distribution of a mean from 9 independent observations. [2 marks]

The variance is $144/9 = 16$.

Answer: $N(80, 16)$.

3. For that sample mean, find P(mean>84). [2 marks]

Standard error is 4; $z = (84 - 80)/4 = 1$.

Answer: 0.1587.

4. If X has variance 4, compare Var(2X) with the variance of X1+X2 for independent copies. [2 marks]

$\text{Var}(2X) = 4(4) = 16$.

Independent sum variance is $4 + 4 = 8$.

Answer: 16 versus 8.

5. Two normally distributed processing times come from the same machine cycle and are strongly related. Can you automatically add their variances? [2 marks]

Dependence may produce nonzero covariance; independence was not established.

Answer: No.

6. Independent normal X,Y have means 20,12 and variances 9,4. Find P(X-Y<5). [3 marks]

$X - Y \sim N(8, 13)$.

Standardise 5 using SD $\sqrt{13}$.

Answer: 0.2027.