Marking guide | Continuous probability and normal distributions

Practice test 3 - applications and assumptions

10 questions | 42 marks | 65 minutes

Connect polynomial densities with practical normal models and compare totals, averages and model choices.

Equivalent correct methods earn the corresponding marks.

1. A proposed density is $f(x) = kx(1-x)$ on [0,1]. [4 marks]

(a) Find k and justify nonnegativity. [2 marks]

$\int_0^1 x(1-x)\,dx = 1/6$; both factors are nonnegative on [0,1].

Answer: $k = 6$.

(b) Find P(0.2<X<0.8). [2 marks]

Use antiderivative $3x^2 - 2x^3$; values are 0.896 and 0.104.

Answer: 0.792.

2. For density $6x(1-x)$ on [0,1], [5 marks]

(a) find E(X). [2 marks]

By symmetry around 1/2, or integrate $6x^2(1-x)$ to obtain 1/2.

Answer: 1/2.

(b) find its variance. [3 marks]

$E(X^2) = 6(1/4 - 1/5) = 3/10$.

$3/10 - (1/2)^2 = 1/20$.

Answer: 1/20.

3. Its interior CDF is $F(x) = 3x^2 - 2x^3$. [4 marks]

(a) Check the two endpoint CDF values. [2 marks]

Direct substitution gives the cumulative limits.

Answer: F(0)=0 and F(1)=1.

(b) Explain why 0.5 is a median and find P(X>0.5). [2 marks]

$3(1/4) - 2(1/8) = 1/2$; take the complement.

Answer: F(0.5)=0.5; upper probability 0.5.

4. A normal model describes delivery time with mean 36 minutes and SD 4 minutes. [4 marks]

(a) Find the chance a delivery takes between 30 and 40 minutes. [2 marks]

The standardised bounds are -1.5 and 1.

Answer: 0.7745.

(b) Find a time exceeded by only 5% of deliveries. [2 marks]

Use lower-tail area 0.95, not 0.05.

Answer: 42.5794 minutes.

5. A normal score distribution has 16th percentile 62 and 84th percentile 78. [5 marks]

(a) Find the mean. [2 marks]

The two percentiles are symmetric, so average the endpoints.

Answer: 70.

(b) Calculate the SD using the exact standard-normal percentiles. [3 marks]

$78 = 70 + \Phi^{-1}(0.84)\sigma$.

Answer: 8.0446.

6. Independent masses $X \sim N(30, 4)$ and $Y \sim N(20, 9)$ are measured in grams. [4 marks]

(a) Find the distribution of 2X+Y. [2 marks]

Mean $2(30) + 20 = 80$; variance $4(4) + 9 = 25$.

Answer: $N(80, 25)$.

(b) Find P(2X+Y>90). [2 marks]

Use the mean 80 and SD 5 to obtain z=2.

Answer: 0.0228

7. Nine independently measured cable lengths each follow a normal model with mean 2 m and SD 0.1 m. [4 marks]

(a) Find the mean and SD of their total length. [2 marks]

Mean is 9(2); variance is $9(0.01) = 0.09$.

Answer: 18 m and 0.3 m.

(b) Find the chance the total is at least 18.3 m. [2 marks]

The standardised cut-off is 1.

Answer: 0.1587

8. The same nine cable lengths are averaged. [4 marks]

(a) Find the standard error of their mean. [2 marks]

$0.1/\sqrt{9} = 0.1/3$.

Answer: $1/30 \approx 0.03333$ m.

(b) Explain why adding 0.3 m to the total corresponds to adding only 1/30 m to the average. [2 marks]

The event total>18.3 is the same as mean>18.3/9, whose bound is 2+1/30.

Answer: The average divides the total by nine.

9. A farm models a nonnegative quantity as normal with mean 1 and SD 2. [4 marks]

(a) Find the normal-model probability of a negative value. [2 marks]

Standardise zero: $(0-1)/2=-0.5$.

Answer: 0.3085

(b) What does that result suggest about the model? [2 marks]

It assigns substantial probability to physically impossible negative values; inspect another distribution or the data.

Answer: The normal model may be poor for this quantity.

10. A researcher compares a random sample of 25 observations with a random sample of 100 from the same population. [4 marks]

(a) Compare the standard errors. [2 marks]

$\sqrt{25}/\sqrt{100}=1/2$.

Answer: The n=100 SE is half the n=25 SE.

(b) Explain why this alone does not prove either sample mean is close to the true mean. [2 marks]

An unusual sample remains possible; representativeness and sampling assumptions also matter.

Answer: SE describes repeated-sampling variability, not certainty about one sample.