Quadratics and Parabolas - Applications test

16 questions | 60 marks | 60 minutes

Use quadratic models in area, design, motion and revenue problems, and justify useful answers within stated domains.

Equivalent correct methods earn the corresponding marks.

1. A rectangular garden is 4 m longer than it is wide. Its area is 96 m². Find its width and length. [3 marks]

Let width be $x > 0$. Then $x(x + 4) = 96$.

$(x + 12)(x - 8) = 0$; reject -12.

Answer: 8 m by 12 m.

2. An 8 cm by 12 cm picture has a uniform border. The picture and border together cover 140 cm². Find the border width. [4 marks]

$(8 + 2x)(12 + 2x) = 140$.

$4x^2 + 40x - 44 = 0$, or $(x + 11)(x - 1) = 0$.

Use $x = 1$ because the width is positive.

Answer: 1 cm.

3. Two consecutive positive integers have product 156. Find them. [3 marks]

$n(n + 1) = 156$ gives $(n + 13)(n - 12) = 0$.

Use the positive integer $n = 12$.

Answer: 12 and 13.

4. A ball is modelled by $h = -t^2 + 8t + 2$, where h is metres and t is seconds after release. [4 marks]

(a) Find the release height. [1 mark]

Substitute $t = 0$.

Answer: 2 m.

(b) Find the greatest height and when it occurs. [3 marks]

$h = -(t - 4)^2 + 18$.

Answer: 18 m at $t = 4$ s.

5. For $h = -t^2 + 8t + 2$, find the two non-negative times when the ball is 14 m high. Explain why two times are possible. [4 marks]

$-t^2 + 8t + 2 = 14$ gives $t^2 - 8t + 12 = (t - 2)(t - 6) = 0$.

The times lie on opposite sides of the maximum at $t = 4$.

Answer: $t = 2$ s and $t = 6$ s, once rising and once falling.

6. A rectangular tank is 2 m deep. Its length is 3 m more than its width. It holds 80 m³ when full. Find its width and length. [4 marks]

$2x(x + 3) = 80$, so $(x + 8)(x - 5) = 0$.

Use the positive width $x = 5$.

Answer: Width 5 m; length 8 m.

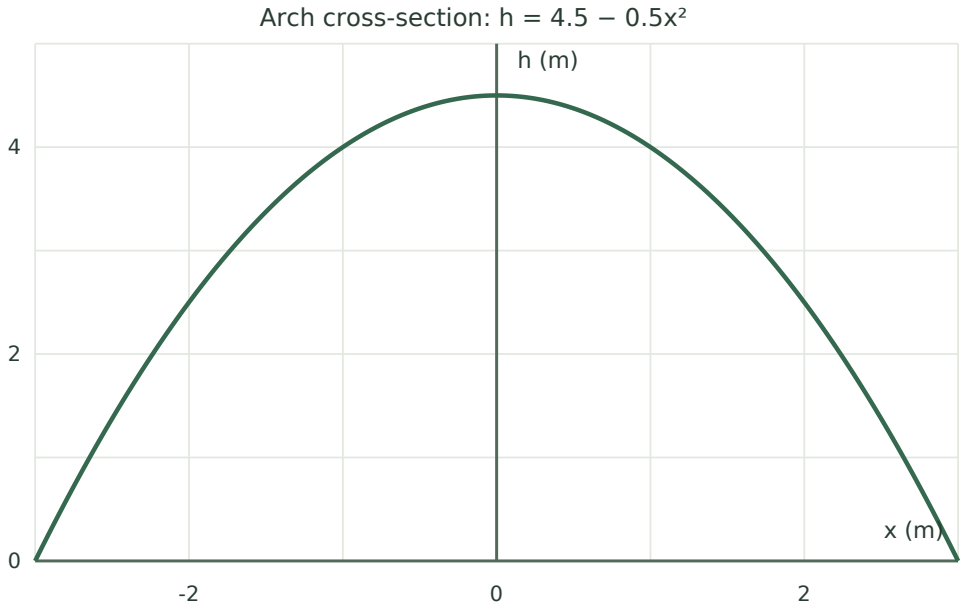
7. A 24 m wire makes a rectangle. Let its width be x metres. Find the greatest possible area and the dimensions giving it. [4 marks]

The length is $12 - x$, so $A = x(12 - x) = -(x - 6)^2 + 36$.

The maximum occurs at $x = 6$; then the length is also 6.

Answer: Greatest area 36 m², dimensions 6 m by 6 m.

8. The arch shown has $h = 4.5 - 0.5x^2$ for $-3 \le x \le 3$. Find its ground-level width and greatest height. [3 marks]

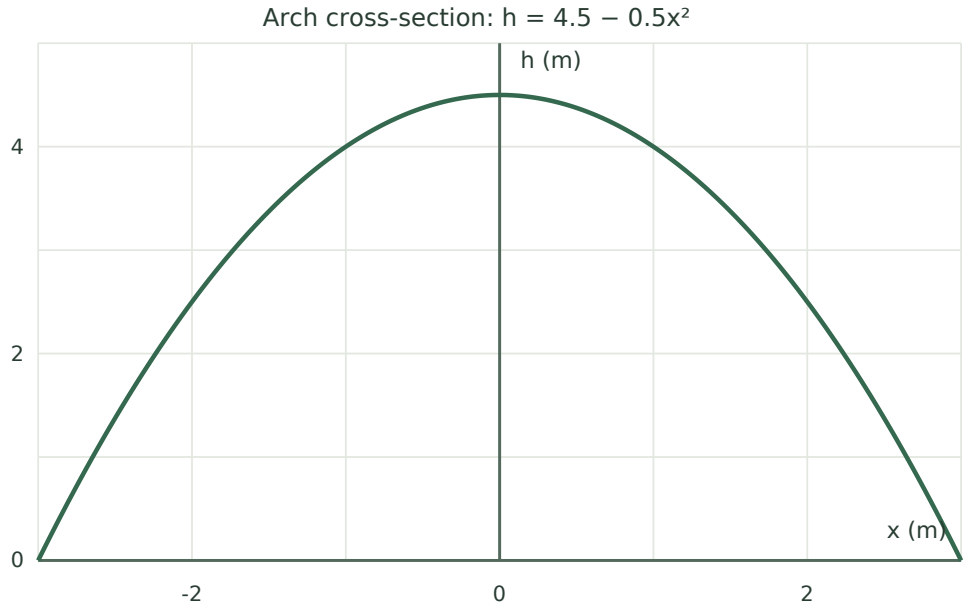


At ground level, $x^2 = 9$, so the endpoints are -3 and 3.

The vertex is $(0, 4.5)$.

Answer: Width 6 m; greatest height 4.5 m.

9. A rectangular truck is 4 m wide and 3 m high. It is centred beneath an arch with $h = 4.5 - 0.5x^2$ metres. Does it fit without touching? Justify using its upper corners. [3 marks]



The edges are at $x = \pm 2$.

$h = 4.5 - 0.5(4) = 2.5 < 3$.

Answer: No. The arch is only 2.5 m high at each truck edge.

10. A stall has modelled profit $P = -n^2 + 16n - 39$ dollars for whole-number n from 0 to 16. For which values of n is the profit positive? [4 marks]

$P = -(n - 3)(n - 13)$.

The downward-opening graph is above zero between roots 3 and 13.

Use whole numbers strictly between these roots.

Answer: $n = 4, 5, \ldots, 12$.

11. A demand model predicts $(60 - p)$ tickets sold at p dollars each, for $0 \le p \le 60$. Find the revenue-maximising price and predicted revenue. [3 marks]

$R = p(60 - p) = -(p - 30)^2 + 900$.

The allowed price $p = 30$ gives 30 tickets.

Answer: Price 30; revenue 900.

12. A rectangular pen against a straight wall uses 40 m of fencing for the other three sides. Let each side perpendicular to the wall have length x metres. Find the greatest area. [4 marks]

The third fenced side is $40 - 2x$.

$A = x(40 - 2x) = -2(x - 10)^2 + 200$.

Answer: Greatest area 200 m²; perpendicular sides 10 m and parallel side 20 m.

13. Two moving objects have height models $h_A = -t^2 + 6t + 1$ and $h_B = -2t^2 + 8t + 1$, in metres. At what time after t=0 are their heights equal, and what is that height? [4 marks]

Set the expressions equal: $t^2 - 2t = t(t - 2) = 0$.

Discard the release time 0. At $t = 2$, both heights are 9.

Answer: $t = 2$ s; common height 9 m.

14. A ball has height $h = -5t^2 + 10t + 1$ metres, where t is seconds. Use the model from release until its first ground contact. [5 marks]

(a) Find the greatest height. [2 marks]

The completed-square form is $-5(t-1)^2 + 6$.

Answer: 6 m.

(b) Find the ground-contact time to two decimal places. [3 marks]

Use the positive root $1 + \sqrt{30}/5 \approx 2.095445$.

Answer: 2.10 s.

15. A rectangle has diagonal 13 cm. Its length is 7 cm more than its width. Use Pythagoras and a quadratic equation to find its sides. [4 marks]

$x^2 + (x+7)^2 = 169$.

$2x^2 + 14x - 120 = 0$, so $(x+12)(x-5) = 0$.

Use the positive width 5.

Answer: 5 cm and 12 cm.

16. A production model is $C = n^2 - 10n + 40$ dollars for $0 \le n \le 12$. A student tries to find its minimum by solving C=0. Explain why this fails and find the true minimum. [4 marks]

$C = (n-5)^2 + 15$, so the minimum is 15.

Equivalently, the equation $C = 0$ has discriminant $100 - 160 = -60$.

A minimum need not be a root.

Answer: The model never reaches zero; its minimum is 15 at $n = 5$.