Marking guide | Probability

Probability - Applications test

16 questions | 48 marks | 60 minutes

Apply compound-event models to games, collections and evidence, then defend assumptions and conclusions.

Equivalent correct methods earn the corresponding marks.

1. A computer model gives success probability 0.3 on each of two independent days. Find P(success on both days). [2 marks]

$0.3 \times 0.3 = 0.09$ under the stated independence assumption.

Answer: 0.09

2. A lunch bundle offers 3 sandwiches, 2 drinks and 2 snacks. Every combination is equally likely. Find P(vegetarian sandwich and water) if exactly one sandwich is vegetarian and exactly one drink is water. [3 marks]

There are $3 \times 2 \times 2 = 12$ bundles.

The preferred sandwich/drink can be paired with either snack: two favourable bundles.

Answer: 1/6

3. A box has five 10-dollar vouchers and three 20-dollar vouchers. Choose two at random without replacement. Find P(two 20-dollar vouchers). [2 marks]

$3/8 \times 2/7 = 6/56 = 3/28$.

Answer: 3/28

4. A game uses a fair die and an independent equal spinner labelled A and B. A player wins only with an even die result and A. [4 marks]

(a) Find the win probability. [2 marks]

$3/6 \times 1/2 = 1/4$.

Answer: 1/4

(b) Predict the wins in 200 plays. Is this count guaranteed? [2 marks]

$200 \times 1/4 = 50$.

Random variation can change the observed count.

Answer: 50 expected wins; not guaranteed.

5. A tray has 8 good parts and 2 faulty parts. Choose two at random without replacement. Find P(no faulty parts). [2 marks]

$8/10 \times 7/9 = 56/90 = 28/45$.

Answer: 28/45

6. A tray has 8 good parts and 2 faulty parts. Choose two at random without replacement. Find P(at least one faulty part). [3 marks]

The complement is two good parts, probability 28/45.

$1 - 28/45 = 17/45$.

Answer: 17/45

7. Among 40 students, 22 used a bus this week, 16 used a bike and 6 used both. Find P(neither) for one randomly selected student. [3 marks]

At least one: $22 + 16 - 6 = 32$.

Neither: $40 - 32 = 8$, so $8/40 = 1/5$.

Answer: 1/5

8. Among 40 students, 22 used a bus this week, 16 used a bike and 6 used both. One student is chosen at random. Find P(exactly one of bus or bike). [3 marks]

Bus only: 16; bike only: 10.

Exactly one: 26; $26/40 = 13/20$.

Answer: 13/20

9. A drawer holds 4 blue and 2 red socks. Two are chosen at random without replacement. Find P(matching colours). [3 marks]

BB gives $4/6 \times 3/5 = 12/30$.

RR gives $2/6 \times 1/5 = 2/30$.

Add: $14/30 = 7/15$.

Answer: 7/15

10. A two-stage game is won 36 times in 90 plays. Use these results to estimate wins in another 120 similar plays. [2 marks]

Estimated probability is $36/90 = 0.4$.

$120 \times 0.4 = 48$.

Answer: About 48 wins.

11. A class simulation records 40 successes in 300 trials. A fair-model probability is 1/8. Compare the observed and model rates. [3 marks]

$40/300 = 2/15$.

The small difference alone does not prove the model wrong.

Answer: Observed $2/15 \approx 0.133$ versus model $1/8 = 0.125$.

12. Group A simulates two draws with replacement; Group B simulates two draws without replacement. Why should their success counts not be pooled as an estimate for one unchanged game? [3 marks]

Replacement changes the second-stage collection.

First choose one game rule and combine only comparable trials.

Answer: They model different rules and can have different event probabilities.

13. Use independent equally likely random digits. Digits 0-3 mean S and 4-9 mean F. Read pairs 12, 74, 06, 39, 88, 45. [6 marks]

(a) Write the six outcomes and estimate P(exactly one S). [3 marks]

Two of six trials have exactly one S.

Answer: SS, FF, SF, SF, FF, FF; estimate 1/3.

(b) Find the theoretical probability of exactly one S. [3 marks]

Each stage has success probability 0.4.

SF and FS have total probability $0.4 \times 0.6 + 0.6 \times 0.4 = 0.48$.

Answer: 0.48

14. A class correctly removes the first counter during each two-draw trial, but never restores the full bag between trials. Explain why this changes the intended simulation. [3 marks]

Keep removal within a trial.

Reset the full starting collection between complete trials.

Answer: Later trials start from a different bag, so they no longer repeat the same experiment.

15. A fair coin chooses bag A or bag B. A has 3 red and 1 blue counter; B has 1 red and 3 blue. Draw one counter at random from the chosen bag. Find P(red). [3 marks]

Red through A: $1/2 \times 3/4 = 3/8$.

Red through B: $1/2 \times 1/4 = 1/8$.

Add disjoint paths to get 1/2.

Answer: 1/2

16. A model predicts win probability 2/5. In 200 plays there are 73 wins. Give a justified short conclusion and one useful next step. [3 marks]

Expected wins: $200 \times 2/5 = 80$.

Observed frequency: $73/200 = 0.365$.

A seven-win difference is not, by itself, proof of a faulty model.

Answer: The observed rate 0.365 is below 0.4, but finite variation may explain the difference; check the rules and repeat more trials.