Statements, implication, converse and equivalence

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. State the assumption and conclusion in: if an integer is divisible by 10, it is divisible by 5. [2 marks]

The condition after “if” is assumed. The statement after “then” must be established.

Answer: Assumption: divisible by 10. Conclusion: divisible by 5.

2. Prove that every multiple of 6 is a multiple of 3. [2 marks]

Write $n = 6k = 3(2k)$ for $k \in \mathbb{Z}$. Since $2k$ is an integer, $3 \mid n$.

Answer: True.

3. Write the converse of the claim in Question 2 and give a counterexample. [2 marks]

3 is divisible by 3 but not by 6.

Answer: Every multiple of 3 is a multiple of 6; false at 3.

4. For real x, test both directions of $x = 2 \Rightarrow x^2 = 4$. [2 marks]

Substitution gives $2^2 = 4$.

For the converse, $x = -2$ has $x^2 = 4$ but $x \neq 2$.

Answer: The original is true; the converse is false.

5. For real x, prove $x = 0 \Leftrightarrow x^2 = 0$. [3 marks]

If $x = 0$, squaring gives 0.

If $x^2 = 0$, then $x \cdot x = 0$, so the zero-product property gives $x = 0$.

Answer: Both directions hold.

6. Is “n is a multiple of 12” sufficient, necessary, both or neither for “n is a multiple of 4”, on the integers? [3 marks]

Every $12k = 4(3k)$ is divisible by 4.

The number 4 is divisible by 4 but not by 12.

Answer: Sufficient but not necessary.