Direct proof and contradiction

6 questions | 16 marks

Equivalent correct methods earn the corresponding marks.

1. Prove that the sum of two even integers is even. [2 marks]

Write $2a + 2b = 2(a + b)$, where $a + b$ is an integer.

Answer: It is even.

2. Prove that the product of two odd integers is odd. [3 marks]

$(2a + 1)(2b + 1) = 2(2ab + a + b) + 1$. The bracket is an integer.

Answer: It is odd.

3. Prove that consecutive integers have an even product. [3 marks]

One of $n, n + 1$ is even, so their product contains a factor of 2.

Answer: $n(n + 1)$ is even.

4. Prove by contradiction that a real number cannot be both positive and negative. [2 marks]

Assume $x > 0$ and $x < 0$. These require x to lie on opposite sides of zero simultaneously, contradicting the order properties of real numbers.

Answer: Impossible.

5. Prove by contradiction that there is no largest integer. [3 marks]

Suppose N is the largest integer. Then $N + 1$ is an integer larger than N, a contradiction.

Answer: No largest integer exists.

6. A proof writes two odd numbers as $2k + 1$ and $2k + 1$. Explain the gap and repair it. [3 marks]

Use $2a + 1$ and $2b + 1$ with independent integer parameters; the argument then applies to every pair.

Answer: It assumes the odd numbers are equal.