Rational/irrational numbers and simple number proofs

6 questions | 17 marks

Equivalent correct methods earn the corresponding marks.

1. Prove that the sum of two rational numbers is rational. [2 marks]

$ad + bc$ and bd are integers and $bd \neq 0$.

Answer: $a/b + c/d = (ad + bc)/(bd)$.

2. Prove that a non-zero rational multiple of an irrational number is irrational. [3 marks]

Let $r \neq 0$ be rational and x irrational. If rx were rational, then $x = (rx)/r$ would be rational, a contradiction.

Answer: The product is irrational.

3. Show that $0.\overline{27}$ is rational. [3 marks]

Let $x = 0.2727\ldots$. Then $100x - x = 27$, so $x = 27/99 = 3/11$.

Answer: 3/11.

4. Give irrational numbers whose sum is rational, and another pair whose sum is irrational. [2 marks]

0 is rational. A non-zero rational multiple of $\sqrt{2}$ is irrational.

Answer: $\sqrt{2} + (-\sqrt{2}) = 0$; $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$.

5. Prove that $\sqrt{5}$ is irrational. [4 marks]

Assume $\sqrt{5} = a/b$ in lowest terms. Then $a^2 = 5b^2$.

A square divisible by the prime 5 has its root divisible by 5. Set $a = 5k$.

Then $b^2 = 5k^2$, so 5 also divides b, contradicting lowest terms.

Answer: It is irrational.

6. Is the reciprocal of a non-zero irrational number irrational? Prove your answer. [3 marks]

If $1/x$ were rational and non-zero, its reciprocal x would be rational, a contradiction.

Answer: Yes.