Proof and circle geometry - Practice test 1

Name: _____ Date: _____

10 questions | 48 marks | 75 minutes

A balanced paper moving from logical statements and exact number arguments to circle geometry and a final congruence proof.

No calculator needed.

1. Work over the integers. Consider: if n is a multiple of 8, then n is even. [4 marks]

(a) Prove the implication. [2 marks]

(b) Write its converse and disprove that converse. [2 marks]

2. A student claims $x^2 \geq x$ for every real x. [4 marks]

(a) Find a counterexample and show the failed inequality. [2 marks]

(b) Explain why testing x=1,2,3 does not prove the original claim. [2 marks]

3. Let a and b be odd integers. [6 marks]

(a) Prove their sum is even. [3 marks]

(b) Prove their product is odd. [3 marks]

4. Use exact reasoning about rational numbers. [5 marks]

(a) Show $0.\overline{36}$ is rational. [2 marks]

(b) Given that $\sqrt{3}$ is irrational, prove $2-\sqrt{3}$ is irrational. [3 marks]

5. Let $A = [-3, 4)$ and $B = (1, 7]$. [4 marks]

(a) Find $A \cap B$. [2 marks]

(b) Find $A \cup B$. [2 marks]

6. A and B are on a circle with centre O. The minor central angle AOB is 118°. P is on the major arc AB, and Q is on the minor arc AB. [4 marks]

(a) Find angle APB. [2 marks]

(b) Find angle AQB and explain the different arc. [2 marks]

7. ABCD is a cyclic quadrilateral in that order. Angle A is $3x + 5$ degrees and angle C is $2x + 25$ degrees. Angle B is 96°. [4 marks]

(a) Find x and angle A. [2 marks]

(b) Find angle D and check all four angles sum to a full turn. [2 marks]

8. A chord is 8 cm from the centre of a circle of radius 17 cm. [5 marks]

(a) Find the length of the chord. [3 marks]

(b) A second chord is also 30 cm long. State its distance from the centre with a reason. [2 marks]

9. From an external point P, a secant meets a circle at near point A and far point B. PA is 5 cm and AB is 15 cm. PT is tangent. [6 marks]

(a) Find PT. [3 marks]

(b) Another secant has outside segment 4 cm. Find its inside segment. [3 marks]

10. Triangle ABC has AB=AC. D is the midpoint of BC. [6 marks]

(a) Prove triangles ABD and ACD congruent, matching the vertices. [3 marks]

(b) Deduce that AD bisects angle BAC and is perpendicular to BC. [3 marks]