# Proof and circle geometry - Practice test 2

Name: _____ Date: _____

10 questions | 52 marks | 75 minutes

A second balanced paper emphasising converse arguments, divisibility, constructions and similarity.

No calculator needed.

**1.** For a real number x, consider the statement $x = 3 \Rightarrow x^2 = 9$. [5 marks]

**(a)** Explain why the implication is true. [2 marks]

**(b)** Write a corrected equivalence for $x^2 = 9$. [3 marks]

**2.** Consider two geometric claims. [5 marks]

**(a)** Disprove: every quadrilateral with a pair of parallel sides is a parallelogram. [3 marks]

**(b)** Disprove: every rhombus is a square. [2 marks]

**3.** Let n be an integer. [5 marks]

**(a)** Prove $n(n+1)$ is even. [2 marks]

**(b)** Prove $n(n+1)(n+2)$ is divisible by 6. [3 marks]

**4.** Let r be a non-zero rational number and x be irrational. [5 marks]

**(a)** Prove rx is irrational. [3 marks]

**(b)** Explain why the condition $r \neq 0$ is essential. [2 marks]

**5.** Describe solution sets over the reals. [4 marks]

**(a)** Solve $x^2 \leq 16$ and write an interval. [2 marks]

**(b)** Find $[-4, 4] \cap (2, \infty)$. [2 marks]

**6.** AB is a diameter and P is another point on its circle. AP is 7 cm and PB is 24 cm. [5 marks]

**(a)** Find AB. [3 marks]

**(b)** A student calls AP the hypotenuse. Explain the error using the angle location. [2 marks]

**7.** ABCD is an isosceles trapezium with AB parallel to CD, and base angles A and B equal. [5 marks]

**(a)** Explain why angle A plus angle D is $180^\circ$. [2 marks]

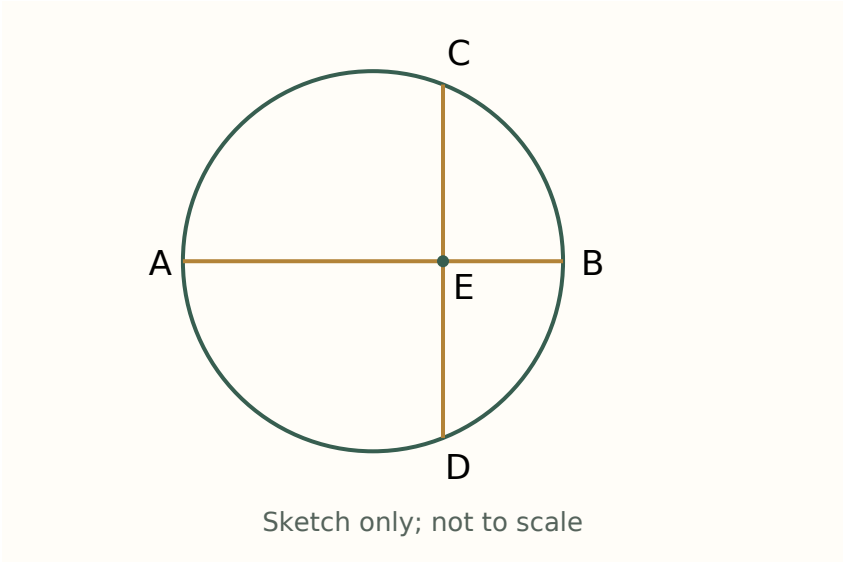
**(b)** Prove the trapezium is cyclic. [3 marks]

**8.** Tangents PA and PB touch a circle with centre O. The angle APB is $70^\circ$. [6 marks]

**(a)** Find angle AOB. [3 marks]

**(b)** Explain why OP bisects angle APB. [3 marks]

**9.** Chords AB and CD meet at E. AE=x, EB=x+5, CE=3 and ED=12, in centimetres. [5 marks]




Sketch only; not to scale

**(a)** Form and solve an equation for x. [3 marks]

**(b)** State the permitted x and find AB. [2 marks]

**10.** In a right triangle ABC, angle A is right and altitude AH meets BC. BH=9 cm and HC=16 cm. [7 marks]

**(a)** Find AH using the geometric-mean relationship. [3 marks]

**(b)** Find AB and AC, explaining which hypotenuse piece belongs to each. [4 marks]