Solving trig equations and describing all solutions

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Solve $\cos x = -1/2$ on $[0, 2\pi)$. [2 marks]

Reference angle $\pi/3$; use quadrants II and III.

Answer: $2\pi/3, 4\pi/3$.

2. Solve $\sin x = 0$ on $[-\pi, \pi]$. [2 marks]

All integer multiples of π in the closed interval count.

Answer: $-\pi, 0, \pi$.

3. Give all real solutions of $\cos x = 1$. [2 marks]

Cosine is 1 at complete turns.

Answer: $x = 2k\pi,\ k \in \mathbb{Z}$.

4. Solve $2\cos^2 x - \cos x - 1 = 0$ on $[0, 2\pi)$. [2 marks]

Factor $(2\cos x + 1)(\cos x - 1) = 0$; solve cosine $-1/2$ or 1.

Answer: $0, 2\pi/3, 4\pi/3$.

5. Solve $\sin(2x) = 0$ on $[0, \pi]$. [2 marks]

$2x = k\pi$ with $k = 0, 1, 2$.

Answer: $0, \pi/2, \pi$.

6. Solve $\sin x(2\cos x - 1) = 0$ on $[0, 2\pi)$. [2 marks]

Sine zero gives 0 and π; cosine $1/2$ gives the other two.

Answer: $0, \pi/3, \pi, 5\pi/3$.