Compound-angle identities

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find $\sin 15^\circ$ exactly. [2 marks]

Expand $\sin(45^\circ - 30^\circ)$.

Answer: $(\sqrt{6} - \sqrt{2})/4$.

2. Find $\cos 75^\circ$ exactly. [2 marks]

Expand $\cos(45^\circ + 30^\circ)$.

Answer: $(\sqrt{6} - \sqrt{2})/4$.

3. Find $\tan 75^\circ$ exactly. [2 marks]

Use $(1 + 1/\sqrt{3})/(1 - 1/\sqrt{3})$, then rationalise.

Answer: $2 + \sqrt{3}$.

4. For acute A, B with $\sin A = 5/13$ and $\sin B = 3/5$, find $\sin(A + B)$. [2 marks]

The missing cosines are $12/13$ and $4/5$.

Add $20/65 + 36/65$.

Answer: $56/65$.

5. Given $\tan A = 2$, $\tan B = 1/3$, calculate $\tan(A - B)$. [2 marks]

$(2 - 1/3)/(1 + 2/3) = (5/3)/(5/3) = 1$.

Answer: 1.

6. Disprove $\cos(A + B) = \cos A + \cos B$. [2 marks]

Both sides are defined, and the unequal values disprove the universal claim.

Answer: Set $A = B = 0$: the two sides are 1 and 2.