Periodic modelling

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. A model has maximum $28\,^\circ\text{C}$ and minimum $16\,^\circ\text{C}$. Find amplitude and midline. [2 marks]

Half-difference is 6; average is 22.

Answer: Amplitude $6\,^\circ\text{C}$; midline $22\,^\circ\text{C}$.

2. A wheel has radius 8 m, axle 10 m above ground and period 40 s. It starts at the bottom. Give a height rule. [2 marks]

The midline is 10, amplitude 8 and angular frequency $\pi/20$; negative cosine begins at height 2.

Answer: $h(t) = 10 - 8\cos(\pi t/20)$ m.

3. For that wheel, find the height after 10 s. [2 marks]

The argument is $\pi/2$, whose cosine is 0.

Answer: 10 m.

4. For that wheel, find every time at height 14 m on $[0, 40)$. [2 marks]

Solve cosine $-1/2$, giving arguments $2\pi/3, 4\pi/3$.

Multiply by $20/\pi$.

Answer: $t = 40/3, 80/3$ s.

5. Why is $\cos(2\pi t/12)$ inappropriate for a 365-day daylight cycle when t is in days? [2 marks]

A 365-day cycle needs angular frequency $2\pi/365$ per day.

Answer: It repeats after 12 days.

6. Give two limitations of modelling a tide with a fixed sine wave. [2 marks]

The fixed amplitude/period assumptions ignore those effects, so predictions must be checked.

Answer: Weather can change heights; real successive tidal cycles can have unequal highs or timing.