Further trigonometry - Practice test 1

10 questions | 42 marks | 65 minutes

Build from exact angles and graph features to identities and periodic applications. Angles are in radians unless a degree symbol is shown.

Equivalent correct methods earn the corresponding marks.

1. Work with radian measure. [4 marks]

(a) Convert 300° to radians. [2 marks]

$300\pi/180 = 5\pi/3$.

Answer: $5\pi/3$.

(b) A sector has radius 4 cm and angle $5\pi/3$. Find its area. [2 marks]

$A = \frac{1}{2}(4^2)(5\pi/3) = 40\pi/3$.

Answer: $40\pi/3$ cm^2.

2. Consider $f(x) = 2 - 3\cos(2x)$. [4 marks]

(a) State its amplitude, period and range. [2 marks]

Use $|A| = 3$, $T = 2\pi/2$ and midline 2.

Answer: Amplitude 3; period π; range $[-1, 5]$.

(b) Find the first maximum at $x \geq 0$. [2 marks]

A maximum needs $\cos 2x = -1$, first at $2x = \pi$.

Answer: $x = \pi/2$, $f(x) = 5$.

3. Solve and check the solution interval. [4 marks]

(a) Solve $\sin x = -1/2$ on $[0, 2\pi)$. [2 marks]

Sine is negative in quadrants III and IV; reference angle $\pi/6$.

Answer: $7\pi/6, 11\pi/6$.

(b) Give all real solutions of $\tan(2x) = \sqrt{3}$. [2 marks]

$2x = \pi/3 + k\pi$; divide by 2.

Answer: $x = \pi/6 + k\pi/2$, $k \in \mathbb{Z}$.

4. Use compound-angle formulas. [4 marks]

(a) Find $\cos 15^\circ$ exactly. [2 marks]

Use $\cos(45^\circ - 30^\circ) = \cos 45 \cos 30 + \sin 45 \sin 30$.

Answer: $(\sqrt{6} + \sqrt{2})/4$.

(b) For acute A, B with $\sin A = 4/5$ and $\cos B = 5/13$, find $\sin(A - B)$. [2 marks]

$\cos A = 3/5$, $\sin B = 12/13$.

Compute $(4/5)(5/13) - (3/5)(12/13) = -16/65$.

Answer: $-16/65$.

5. Let $\tan\theta = 3/4$ with θ acute. [4 marks]

(a) Find $\sin 2\theta$ and $\cos 2\theta$. [2 marks]

Use a 3-4-5 triangle: sine 3/5, cosine 4/5.

Then $2(3/5)(4/5) = 24/25$ and $16/25 - 9/25 = 7/25$.

Answer: 24/25 and 7/25.

(b) Find $\tan 2\theta$. [2 marks]

Divide the double-angle sine by cosine, or use $2(3/4)/(1 - 9/16)$.

Answer: 24/7.

6. Let $g(x) = 5 + 12\sin x + 5\cos x$. [4 marks]

(a) Express the varying part as one sine, specifying an acute phase. [2 marks]

Amplitude $\sqrt{144 + 25} = 13$; match cosine of phase 12/13 and sine 5/13.

Answer: $13\sin(x + \phi)$, $\phi = \tan^{-1}(5/12)$.

(b) State the range of g. [2 marks]

Translate the interval $[-13, 13]$ by 5.

Answer: $[-8, 18]$.

7. Consider reciprocal trigonometry. [4 marks]

(a) Find all excluded inputs for $y = \sec(3x)$. [2 marks]

Exclude $3x = \pi/2 + k\pi$.

Answer: $x = \pi/6 + k\pi/3$.

(b) Solve $\csc x = -2$ on $[0, 2\pi)$. [2 marks]

Equivalent to $\sin x = -1/2$; neither solution is excluded.

Answer: $7\pi/6, 11\pi/6$.

8. Prove an identity and preserve its domain. [5 marks]

(a) Show $(1 - \cos 2x)/(1 + \cos 2x) = \tan^2 x$. [3 marks]

Use numerator $2\sin^2 x$ and denominator $2\cos^2 x$; divide.

Answer: The ratio is $\tan^2 x$ wherever $\cos x \neq 0$.

(b) State every excluded input from the original ratio. [2 marks]

$1 + \cos 2x = 2\cos^2 x$ is zero exactly at those inputs.

Answer: $x = \pi/2 + k\pi$.

9. A model temperature is $T(t) = 18 + 6\cos(\pi(t-14)/12)$ degrees Celsius, where t is hours after midnight. [4 marks]

(a) Find the maximum and minimum temperatures and their first occurrence in $[0, 24)$. [2 marks]

Cosine is 1 at 14; it is -1 half a 24-hour period earlier at 2.

Answer: Maximum 24 at $t = 14$; minimum 12 at $t = 2$.

(b) Find $T(8)$ and explain one modelling assumption. [2 marks]

The argument at 8 is $-\pi/2$, whose cosine is 0.

Weather may change the actual cycle.

Answer: $18\,^{\circ}\text{C}$; the model assumes a fixed repeating daily cycle.

10. Solve an equation without dividing away a factor. [5 marks]

(a) Solve $2\sin x\cos x = \sin x$ on $[0, 2\pi)$. [3 marks]

Factor $\sin x(2\cos x - 1) = 0$.

Sine zero gives 0 and π; cosine 1/2 gives the other two.

Answer: $0, \pi/3, \pi, 5\pi/3$.

(b) Explain what would be lost by immediately dividing by $\sin x$. [2 marks]

At these values the factor being divided by is zero, although the original equation is true.

Answer: The valid solutions 0 and π.