Further trigonometry - Practice test 2

10 questions | 40 marks | 65 minutes

Use representations, exact reasoning and graph restrictions. Supply exact answers where possible and state domains.

Equivalent correct methods earn the corresponding marks.

1. A point on the unit circle lies in quadrant III and has horizontal coordinate $-12/13$. [4 marks]

(a) Find its vertical coordinate. [2 marks]

Square and add to 1: the other coordinate has magnitude 5/13.

Quadrant III makes it negative.

Answer: $-5/13$.

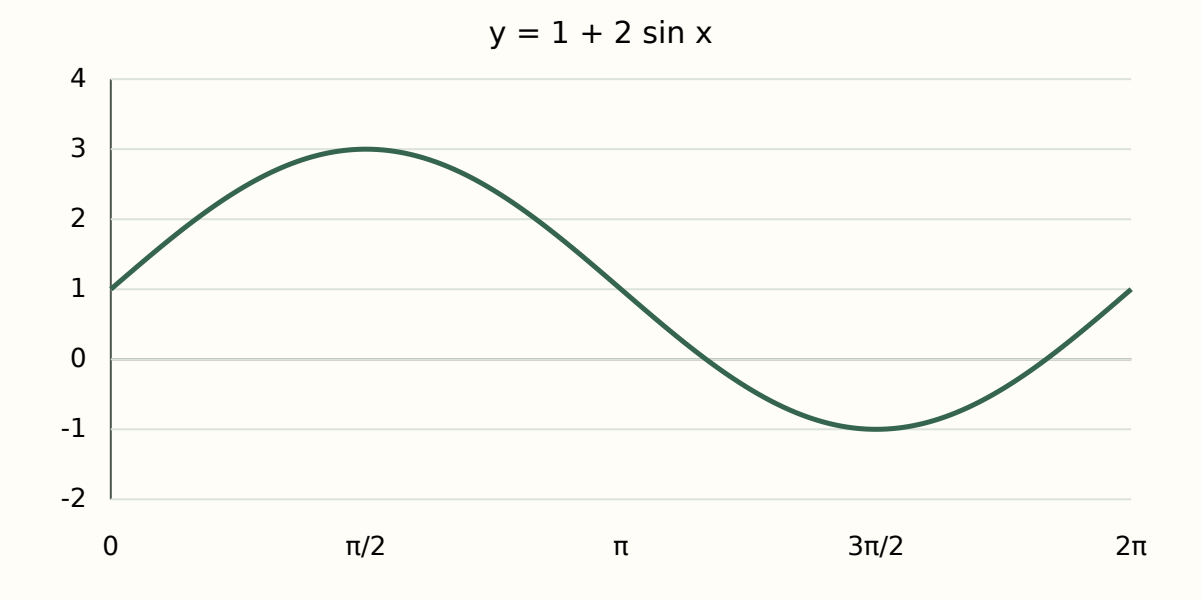
(b) Find $\tan\theta$ and $\cos(2\theta)$. [2 marks]

Tangent is $(-5/13)/(-12/13)$.

Double-angle cosine is $2(144/169) - 1 = 119/169$.

Answer: $5/12$ and $119/169$.

2. The graph below shows $y = 1 + 2\sin x$. [4 marks]



(a) State the midline and range. [2 marks]

The amplitude is 2 around height 1.

Answer: Midline $y = 1$; range $[-1, 3]$.

(b) Give the coordinates of both turning points on $[0, 2\pi]$. [2 marks]

Sine is 1 at $\pi/2$ and -1 at $3\pi/2$.

Answer: Maximum $(\pi/2, 3)$; minimum $(3\pi/2, -1)$.

3. Solve equations on the interval $0 \le x \le 2\pi$. [4 marks]

(a) Solve $\cos(x/2) = 0$. [2 marks]

The argument is in $[0, \pi]$, where cosine vanishes only at $\pi/2$.

Answer: $x = \pi$.

(b) Solve $\sin^2 x = 1/4$. [2 marks]

Take both signs: $\sin x = \pm 1/2$.

Neither endpoint is a solution.

Answer: $\pi/6, 5\pi/6, 7\pi/6, 11\pi/6$.

4. Develop exact angle values. [4 marks]

(a) Find $\tan 15^\circ$ exactly. [2 marks]

Use $(1 - 1/\sqrt{3})/(1 + 1/\sqrt{3})$ and rationalise.

Answer: $2 - \sqrt{3}$.

(b) Use the sum formula to show $\sin(\pi + x) = -\sin x$. [2 marks]

$\sin\pi\cos x + \cos\pi\sin x = 0 - \sin x$.

Answer: Expand using sine of pi 0 and cosine of pi -1.

5. An angle α lies in quadrant IV with $\cos\alpha = 5/13$. [4 marks]

(a) Find $\sin 2\alpha$. [2 marks]

Sine is $-12/13$; use $2\sin\alpha\cos\alpha$.

Answer: $-120/169$.

(b) Find $\cos^2\alpha - \sin^2\alpha$ without calculating the angle. [2 marks]

Use $(25 - 144)/169$ or $2\cos^2\alpha - 1$.

Answer: $-119/169$.

6. Work with $u(x) = \sqrt{3}\cos x - \sin x$. [4 marks]

(a) Write u as one cosine with positive amplitude. [2 marks]

Expand: $2\cos x(\sqrt{3}/2) - 2\sin x(1/2)$.

Answer: $2\cos(x + \pi/6)$.

(b) Solve $u(x) = 0$ on $[0, 2\pi)$. [2 marks]

Cosine zero requires $x + \pi/6 = \pi/2, 3\pi/2$ in the appropriate shifted interval.

Answer: $x = \pi/3, 4\pi/3$.

7. The function $v(x) = \csc x$ is defined only when $\sin x \neq 0$. [4 marks]

(a) State its vertical asymptotes and range. [2 marks]

Zeros of the sine denominator give asymptotes; nonzero sine has magnitude at most 1.

Answer: Asymptotes $x = k\pi$; range $(-\infty, -1] \cup [1, \infty)$.

(b) Explain why $\csc x = 1/2$ has no real solution. [2 marks]

Sine never exceeds 1, so the reciprocal equation is impossible.

Answer: It would require $\sin x = 2$.

8. Study a proposed identity. [4 marks]

(a) Prove $(1+\sin x)/\cos x = \sec x + \tan x$ on the common domain. [2 marks]

Require $\cos x \neq 0$, then use the definitions.

Answer: Split the fraction into $1/\cos x + \sin x/\cos x$.

(b) Evaluate $(\sec x + \tan x)(\sec x - \tan x)$ and state the same restriction. [2 marks]

Use difference of squares and $\sec^2 x - \tan^2 x = 1$.

Answer: 1 for $\cos x \neq 0$.

9. A machine arm has horizontal coordinate $X(t) = 3\cos(\pi t/4)$ cm, where time is in seconds. [4 marks]

(a) Find its period and the first time it reaches its minimum coordinate. [2 marks]

The period is $2\pi/(\pi/4) = 8$; cosine reaches -1 at argument π.

Answer: Period 8 s; first minimum at 4 s.

(b) Find every t in $[0, 8)$ for which $X(t) = 1.5$ cm. [2 marks]

Cosine equals 1/2 at arguments $\pi/3$ and $5\pi/3$; multiply by $4/\pi$.

Answer: $t = 4/3, 20/3$ s.

10. A student says the amplitude of $6\sin(2x) + 8\cos(2x)$ is 14. [4 marks]

(a) Find the correct amplitude and explain the error. [2 marks]

Same-frequency combination gives $\sqrt{36+64} = 10$.

Answer: 10, because the two terms do not peak together.

(b) State the period and full range after adding a constant -3. [2 marks]

Frequency 2 gives period π; translate $[-10, 10]$ down 3.

Answer: Period π; range $[-13, 7]$.