Further trigonometry - Practice test 4

10 questions | 43 marks | 65 minutes

Use exact calculation and periodic models in unfamiliar but fully specified situations. Interpret all units and restrictions.

Equivalent correct methods earn the corresponding marks.

1. A circular track has radius 12 m. A runner moves through an angle of $7\pi/12$. [4 marks]

(a) Find the distance travelled along the track. [2 marks]

Use $s = r\theta = 12(7\pi/12)$.

Answer: 7π m.

(b) Find the area of the corresponding sector. [2 marks]

Use $\frac{1}{2}(12^2)(7\pi/12) = 42\pi$.

Answer: 42π m^2.

2. A graph is modelled by $p(x) = -2 + 4\cos(3x + \pi)$. [4 marks]

(a) Find its period, horizontal shift and range. [2 marks]

Factor $3x + \pi = 3(x + \pi/3)$; midline -2 and amplitude 4.

Answer: Period $2\pi/3$; left shift $\pi/3$; range $[-6, 2]$.

(b) Give an equivalent rule with no phase shift. [2 marks]

Use $\cos(u + \pi) = -\cos u$.

Answer: $p(x) = -2 - 4\cos 3x$.

3. Solve $\tan(x - \pi/6) = 1$ on $0 \le x < 2\pi$. [4 marks]

(a) Give every solution. [2 marks]

The argument equals $\pi/4 + k\pi$; add $\pi/6$ and select two values.

Answer: $x = 5\pi/12, 17\pi/12$.

(b) State the family of excluded inputs of the original tangent expression. [2 marks]

The argument cannot equal $\pi/2 + k\pi$.

Answer: $x = 2\pi/3 + k\pi$.

4. Acute angles A, B satisfy $\tan A = 1/2$ and $\tan B = 1/3$. [4 marks]

(a) Find $\tan(A + B)$ exactly. [2 marks]

$(1/2 + 1/3)/(1 - 1/6) = (5/6)/(5/6) = 1$.

Answer: 1.

(b) Find $A + B$ exactly, justifying the selected angle. [2 marks]

The sum of two acute angles lies in $(0, \pi)$; tangent 1 there occurs only at $\pi/4$.

Answer: $\pi/4$.

5. Work with double-angle identities. [4 marks]

(a) Show that $4\sin^2 x\cos^2 x = \sin^2 2x$. [2 marks]

Both sides are squares, so the result holds for every real x.

Answer: Square $\sin 2x = 2\sin x\cos x$.

(b) Find every x in $[0, \pi)$ for which $4\sin^2 x\cos^2 x = 1$. [2 marks]

The identity gives $\sin^2 2x = 1$, so $2x = \pi/2, 3\pi/2$ in $[0, 2\pi)$.

Answer: $x = \pi/4, 3\pi/4$.

6. Two same-frequency signals combine as $S(x) = 9\cos x + 12\sin x$. [4 marks]

(a) Find the largest possible value of S and express S as one cosine. [2 marks]

Match cosine coefficient $15\cos\phi = 9$ and sine coefficient $15\sin\phi = 12$.

Answer: 15; $S = 15\cos(x - \phi)$ with $\phi = \tan^{-1}(4/3)$.

(b) Find the first positive input giving this largest value. [2 marks]

The cosine argument is zero at $x = \phi$; ϕ is acute.

Answer: $x = \tan^{-1}(4/3)$.

7. A student proposes that $\sec x$ has a zero at $x = \pi/2$. [4 marks]

(a) Explain the error using the definition. [2 marks]

Its denominator cosine is zero; the numerator 1 never vanishes.

Answer: Secant is undefined there, not zero.

(b) Solve $2\cot x = \sqrt{3}$ on $[0, \pi)$. [2 marks]

Cotangent $\sqrt{3}/2$ gives tangent $2/\sqrt{3}$.

The solution is in quadrant I, and tangent has period π.

Answer: $x = \tan^{-1}(2/\sqrt{3}) \approx 0.857$ rad.

8. Prove $\cos x/(1 + \sin x) = (1 - \sin x)/\cos x$ on the common domain. [5 marks]

(a) Give a valid one-sided proof. [3 marks]

For $\cos x \neq 0$, multiply numerator/denominator by $1 - \sin x$.

The denominator becomes $1 - \sin^2 x = \cos^2 x$; cancel one cosine.

Answer: Multiply the left side by the conjugate and simplify.

(b) Give a point where the original left expression exists but the right expression does not. [2 marks]

At $\pi/2$, the left is $0/(1 + 1) = 0$, while the right is $0/0$ and undefined.

Answer: $x = \pi/2$.

9. An ideal observation wheel has axle height 15 m, radius 10 m and period 60 s. A seat starts at its highest point. [5 marks]

(a) Write a height model in metres. [2 marks]

The midline 15, amplitude 10 and frequency $2\pi/60$ match the context.

Answer: $h(t) = 15 + 10\cos(\pi t/30)$.

(b) For how long in one turn is the seat at least 20 m above ground? [3 marks]

Cosine must be at least 1/2, giving $0 \le t \le 10$ and $50 \le t \le 60$.

The interval lengths add to 20.

Answer: 20 seconds in total.

10. A sinusoid has form $f(x) = D + A\sin x + B\cos x$, with $f(0) = 5$, $f(\pi) = 1$ and $f(\pi/2) = 7$. [5 marks]

(a) Find D, A, B. [3 marks]

The first two observations give $D + B = 5$, $D - B = 1$, so $D = 3$, $B = 2$.

The third gives $D + A = 7$.

Answer: $D = 3$, $A = 4$, $B = 2$.

(b) Find its exact range. [2 marks]

Resultant amplitude is $\sqrt{4^2 + 2^2} = 2\sqrt{5}$.

Answer: $[3 - 2\sqrt{5}, 3 + 2\sqrt{5}]$.