Vector geometry proofs

6 questions | 19 marks

Equivalent correct methods earn the corresponding marks.

1. A, B have position vectors a, b. Write the midpoint M position vector. [2 marks]

Average the two position vectors.

Answer: $\frac{1}{2}(\mathbf{a}+\mathbf{b})$.

2. A, B, C have position vectors a, b, c. M and N are midpoints of AB and AC. Show that MN=$\frac{1}{2}$ BC. [3 marks]

OM=$(\mathbf{a}+\mathbf{b})/2$ and ON=$(\mathbf{a}+\mathbf{c})/2$.

Subtract OM from ON.

Answer: $\overrightarrow{MN}=\frac{1}{2}(\mathbf{c}-\mathbf{b})$.

3. In a parallelogram with adjacent side vectors a and b, show that the diagonals have the same midpoint. [3 marks]

The opposite vertex has position a+b.

The other diagonal joins positions a and b.

Answer: Both midpoints are $\frac{1}{2}(\mathbf{a}+\mathbf{b})$ when one vertex is the origin.

4. For non-zero perpendicular a and b, prove $|\mathbf{a}+\mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2$. [3 marks]

$(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b}) = |\mathbf{a}|^2 + 2\mathbf{a}\cdot\mathbf{b} + |\mathbf{b}|^2$.

The middle term is zero.

Answer: Expand the squared length and use $\mathbf{a}\cdot\mathbf{b} = 0$.

5. A parallelogram has $|\mathbf{a}| = |\mathbf{b}|$. Prove that its diagonal directions a+b and a-b are perpendicular, assuming neither diagonal is zero. [4 marks]

Expand to $|\mathbf{a}|^2 - |\mathbf{b}|^2$.

Equal adjacent lengths make the difference zero.

Answer: $(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}-\mathbf{b}) = 0$.

6. A quadrilateral's diagonals have the same midpoint. Use position vectors a,b,c,d to show AB=DC. [4 marks]

Equal diagonal midpoints give $\mathbf{a}+\mathbf{c} = \mathbf{b}+\mathbf{d}$.

Rearrange to get the two equal directed side vectors.

Answer: $\mathbf{b}-\mathbf{a} = \mathbf{c}-\mathbf{d}$.