# Vectors in two dimensions - Practice test 1

10 questions | 46 marks | 65 minutes

Build from components and direction to projections and a geometric proof.

Equivalent correct methods earn the corresponding marks.

**1.** Let $\mathbf{a} = (3, 4)$ and $\mathbf{b} = (-2, 1)$. [4 marks]

**(a)** Find $2\mathbf{a} - \mathbf{b}$. [2 marks]

$2\mathbf{a} = (6, 8)$; subtract $(-2, 1)$.

**Answer:** $(8, 7)$.

**(b)** Find $|\mathbf{a}|$ and a unit vector in its direction. [2 marks]

$|\mathbf{a}| = \sqrt{9 + 16} = 5$; divide by five.

**Answer:** 5 and $(3/5, 4/5)$.

**2.** A=(1,-2), B=(7,6) and C=(-2,5). [4 marks]

**(a)** Find AB and its length. [2 marks]

AB=$(7 - 1, 6 + 2) = (6, 8)$; length 10.

**Answer:** $(6, 8)$; 10.

**(b)** Find D if CD=BA. [2 marks]

BA=$(-6, -8)$; add to C.

**Answer:** $D = (-8, -3)$.

**3.** The vector $\mathbf{u} = (-5, 5)$ describes a displacement in kilometres. [4 marks]

**(a)** Find its exact magnitude and its direction anticlockwise from the positive x-axis. [2 marks]

Magnitude $\sqrt{25+25} = 5\sqrt{2}$; the direction is quadrant II.

**Answer:** $5\sqrt{2}$ km and $135^\circ$.

**(b)** Find the displacement directly back to the start. [2 marks]

Negate the original displacement.

**Answer:** $(5, -5)$ km.

**4.** A=(2,1), B=(10,5). [4 marks]

**(a)** Find the point P for AP:PB=3:1. [2 marks]

AB=(8, 4); add three quarters of it to A.

**Answer:** (8, 4).

**(b)** Find Q beyond B such that $\overrightarrow{AQ} = \frac{3}{2}\overrightarrow{AB}$. [2 marks]

A+$\frac{3}{2}$(8, 4) = (2, 1) + (12, 6).

**Answer:** (14, 7).

**5.** Let $\mathbf{p} = (2, 3)$ and $\mathbf{q} = (8, k)$. [4 marks]

**(a)** Find k if p and q are parallel. [2 marks]

The first component is multiplied by four, so the second is too.

**Answer:** $k = 12$.

**(b)** Using your k, state whether p and q have the same or opposite directions. [2 marks]

$\mathbf{q} = 4\mathbf{p}$, a positive multiple.

**Answer:** Same direction.

**6.** Let $\mathbf{u} = (1, 2)$ and $\mathbf{v} = (2, -1)$. [4 marks]

**(a)** Calculate $\mathbf{u} \cdot \mathbf{v}$ and state the angle between them. [2 marks]

$1(2) + 2(-1) = 0$; both vectors are non-zero.

**Answer:** $0$ and $90^\circ$.

**(b)** Find $|\mathbf{u} + \mathbf{v}|$ and check it against $|\mathbf{u}|^2 + |\mathbf{v}|^2$. [2 marks]

The sum is $(3, 1)$ with squared magnitude ten.

**Answer:** $\sqrt{10}$; its square is $10 = 5 + 5$.

**7.** Let $\mathbf{a} = (4, 2)$ and $\mathbf{b} = (1, 1)$. [5 marks]

**(a)** Find the scalar projection of a onto b. [2 marks]

$\mathbf{a} \cdot \mathbf{b} = 6$; divide by $\sqrt{2}$.

**Answer:** $3\sqrt{2}$.

**(b)** Find the vector projection and the perpendicular remainder. [3 marks]

Projection coefficient $6/2 = 3$; subtract $(3, 3)$ from a.

**Answer:** Projection $(3, 3)$; remainder $(1, -1)$.

**8.** A force vector is $\mathbf{F} = (6, -3)$ N. The unit direction along a rail is $\mathbf{e} = (2/\sqrt{5}, 1/\sqrt{5})$. [5 marks]

**(a)** Find the signed component of F along the rail. [2 marks]

Dot F with the supplied unit vector: $(12 - 3)/\sqrt{5}$.

**Answer:** $9/\sqrt{5}$ N.

**(b)** Find the component vector along the rail. [3 marks]

Multiply $9/\sqrt{5}$ by e.

**Answer:** $(18/5, 9/5)$ N.

**9.** A=(0,0), B=(4,1), C=(3,5). [6 marks]

**(a)** Show that angle ABC is a right angle. [3 marks]

Both vectors start at B and are non-zero.

**Answer:** BA·BC=$(-4,\ -1)\cdot(-1,4)=0$.

**(b)** Find the exact area of triangle ABC. [3 marks]

The perpendicular sides each have length $\sqrt{17}$.

Area=$\frac{1}{2}\sqrt{17}\sqrt{17}=17/2$.

**Answer:** $17/2$ square units.

**10.** In triangle OAB, OA=a and OB=b. M and N are the midpoints of OA and OB. [6 marks]

**(a)** Express MN in terms of a and b. [3 marks]

ON=$\frac{1}{2}b$ and OM=$\frac{1}{2}a$; subtract.

**Answer:** $\frac{1}{2}(\mathbf{b}-\mathbf{a})$.

**(b)** Prove that MN is parallel to AB and half its length. [3 marks]

AB=b−a; the positive factor $1/2$ preserves direction and halves magnitude.

**Answer:** $\overrightarrow{MN}=\frac{1}{2}\overrightarrow{AB}$, proving both statements.