Vectors in two dimensions - Practice test 2

Name: _____ Date: _____

10 questions | 48 marks | 65 minutes

Use unknown components, coordinate data and direction tests to justify decisions.

A scientific calculator may be used.

1. A delivery robot visits P, Q and R in that order. Coordinates are in metres. [4 marks]

Recorded positions

Point	x (m)	y (m)
P	-2	1
Q	4	9
R	7	5

(a) Find PQ and QR. [2 marks]

(b) Find the total displacement PR and its exact magnitude. [2 marks]

2. The vectors $(2a + 1, b - 3)$ and $(7, -5)$ are equal. [5 marks]

(a) Find a and b. [2 marks]

(b) Find the unit vector opposite to $(7, -5)$. [3 marks]

3. Two vectors satisfy $2\mathbf{u} + \mathbf{v} = (8, 1)$ and $\mathbf{u} - \mathbf{v} = (1, 5)$. [5 marks]

(a) Find u and v. [3 marks]

(b) Find the magnitude of $3\mathbf{v}$. [2 marks]

4. A vector has components $(k, 3)$ and magnitude 5. [4 marks]

(a) Find every possible value of k. [2 marks]

(b) Which value gives an obtuse angle with $(1, 0)$? Explain. [2 marks]

5. A=(1,2), B=(5,8), C=(9,t). [5 marks]

(a) Find t so that A, B and C are collinear. [3 marks]

(b) For this t, state the ratio AB:BC of lengths. [2 marks]

6. Let $\mathbf{u} = (t, 1)$ and $\mathbf{v} = (2, t - 3)$. [4 marks]

(a) Find t so that u and v are perpendicular. [2 marks]

(b) For this t, find a unit vector in the direction of v. [2 marks]

7. Let $\mathbf{a} = (2, 1)$ and $\mathbf{b} = (-1, 3)$. [5 marks]

(a) Find the angle between a and b to 0.1°. [3 marks]

(b) Find the angle between a and -b. [2 marks]

8. Resolve u=(7,1) relative to the direction v=(2,1). [5 marks]

(a) Find the vector projection of u onto v. [2 marks]

(b) Find the perpendicular component and its magnitude. [3 marks]

9. A boat's velocity relative to still water is $(3, 4)$ km/h. The water current is $(1, -1)$ km/h. [5 marks]

(a) Find the boat's ground velocity and ground speed. [2 marks]

(b) Starting from $(2, -1)$ km, where is the boat after 2 hours if both velocities remain constant? [3 marks]

10. A non-degenerate parallelogram has adjacent side vectors a and b. [6 marks]

(a) Show that the squared diagonal lengths differ by $4\mathbf{a} \cdot \mathbf{b}$. [3 marks]

(b) Hence prove that equal diagonal lengths imply the parallelogram is a rectangle. [3 marks]