Vectors in two dimensions - Practice test 3

Name: _____ Date: _____

10 questions | 52 marks | 70 minutes

Connect exact values, projections and geometric structure, with a short proof to finish.

A scientific calculator may be used.

1. Let $\mathbf{v} = (4\sqrt{3}, -4)$. [4 marks]

(a) Find its magnitude and direction from the positive x-axis. [2 marks]

(b) Find the unit vector in the same direction. [2 marks]

2. In an original coordinate system A=(-3,2) and B=(1,5). A new origin is placed at old coordinates (2,-1), with axes pointing in the same directions. [5 marks]

(a) Give the new coordinates of A and B. [2 marks]

(b) Calculate AB in both systems and explain the agreement. [3 marks]

3. A triangular route starts with displacement a=(2,1) km, then b=(-1,4) km, then c back to the starting point. [5 marks]

(a) Find c. [2 marks]

(b) Find the exact total distance travelled. [3 marks]

4. Let u=(2,1), v=(t,3) and w=(4,s). [4 marks]

(a) Find t if v is parallel to u. [2 marks]

(b) Find s if w is perpendicular to u. [2 marks]

5. A=(1,0), B=(5,2), C=(3,6). [6 marks]

(a) Find angle BAC exactly. [3 marks]

(b) Use a second vector calculation to classify the triangle. [3 marks]

6. A force u=(-4,1) N is resolved along the direction v=(3,0). [4 marks]

(a) Find its signed scalar projection and vector projection along v. [2 marks]

(b) Find the perpendicular component. [2 marks]

7. A line passes through O and has direction v=(2,1). P has position u=(5,3). [6 marks]

(a) Find the position of the perpendicular foot H on the line. [3 marks]

(b) Find the exact shortest distance from P to the line. [3 marks]

8. The angle between $(a, 1)$ and $(1, 1)$ is 45°. [6 marks]

(a) Form an equation involving a using the dot product. [3 marks]

(b) Find a and check the original angle condition. [3 marks]

9. A parallelogram has adjacent sides a=(5,0) and b=(3,4). [6 marks]

(a) Show that it is a rhombus and that its diagonals are perpendicular. [3 marks]

(b) Find its area using half the product of the diagonal lengths. [3 marks]

10. Non-zero vectors a and b satisfy $|a| = |b| = L$ and $a \cdot b = L^2/2$. O, A and B have position vectors 0, a and b. [6 marks]

(a) Prove that OAB is equilateral. [3 marks]

(b) If M is the midpoint of AB, find the length OM. [3 marks]