Vectors in two dimensions - Practice test 4

10 questions | 54 marks | 70 minutes

Apply vectors to routes, forces and geometric decisions; distinguish a calculation from its interpretation.

Equivalent correct methods earn the corresponding marks.

1. A rescue vehicle's displacement is 3 km west and 4 km north. [5 marks]

(a) Write its Cartesian displacement and find the distance from its starting point. [2 marks]

East is positive x and north is positive y.

Use Pythagoras for the magnitude.

Answer: $(-3, 4)$ km; 5 km.

(b) Find its bearing, measured clockwise from north, to 0.1°. [3 marks]

It is $\tan^{-1}(3/4) = 36.869\ldots^\circ$ west of north.

Bearing$=360^\circ - 36.869\ldots^\circ$.

Answer: 323.1°.

2. A=(2,-1), B=(6,5), C=(-3,4). [4 marks]

(a) Find D such that CD=AB. [2 marks]

AB=$(4, 6)$; add that displacement to C.

Answer: $D = (1, 10)$.

(b) Find E such that CE=BA. [2 marks]

Reverse AB to $(-4, -6)$, then add it to C.

Answer: $E = (-7, -2)$.

3. A student claims that $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$ for every pair of vectors. [5 marks]

(a) Give a counterexample using a=(3,0) and b=(0,4). [3 marks]

The sum is $(3, 4)$ with length five.

The individual lengths are three and four.

Answer: The left side is 5, but the right side is 7.

(b) State a condition on non-zero a and b that makes the claimed equality true. [2 marks]

Head-to-tail addition then forms one straight segment without a reversal.

Answer: They point in the same direction.

4. A=(2,-1), B=(8,2) and $\mathbf{p} = \mathbf{a} - \frac{1}{2}(\mathbf{b} - \mathbf{a})$. [6 marks]

(a) Find P and explain where it lies. [3 marks]

AB=$(6, 3)$; subtract half of it from A.

The negative parameter means the opposite direction.

Answer: $P = (-1, -5/2)$, beyond A away from B.

(b) Find the ratio AP:PB of lengths. [3 marks]

AP is half the length of AB, while PB is one and a half times AB.

Answer: $1 : 3$.

5. For this question, work done by a constant force is defined by $W = \mathbf{F} \cdot \mathbf{d}$. Force is measured in newtons and displacement in metres. Let F=(4,3) and d=(2,-1). [5 marks]

(a) Find W and give its unit. [2 marks]

$4(2) + 3(-1) = 5$; newton metres are joules.

Answer: 5 J.

(b) Find the signed component of F along the direction of d. [3 marks]

Divide the dot product five by $|d| = \sqrt{5}$.

Answer: $\sqrt{5}$ N.

6. A vector u satisfies $\mathbf{u} \cdot (1, 1) = 5$ and $\mathbf{u} \cdot (2, -1) = 1$. [5 marks]

(a) Find u. [3 marks]

Write u=(x, y): $x + y = 5$ and $2x - y = 1$.

Adding gives $3x = 6$, then $y = 3$.

Answer: $(2, 3)$.

(b) Find its magnitude and the unit vector opposite to it. [2 marks]

Use Pythagoras, then negate and divide by the length.

Answer: $\sqrt{13}$; $(-2/\sqrt{13}, -3/\sqrt{13})$.

7. Let a=(2,1), b=(-1,2). [5 marks]

(a) Show that a+b and a-b are perpendicular. [2 marks]

Calculate both sum and difference.

Answer: Their dot product is $(1, 3) \cdot (3, -1) = 0$.

(b) Find the angle between a and a+b. [3 marks]

Their dot product is five; their lengths are $\sqrt{5}$ and $\sqrt{10}$.

The cosine is $1/\sqrt{2}$.

Answer: 45°.

8. A=(1,2), B=(5,4) and P=(4,5). H is the perpendicular foot from P to the infinite line AB. [7 marks]

(a) Find H by projecting AP onto AB. [4 marks]

AP=$(3, 3)$, AB=$(4, 2)$; projection coefficient is $18/20 = 9/10$.

Add $(18/5, 9/5)$ to A.

Answer: $H = (23/5, 19/5)$.

(b) Find PH and explain whether H is on segment AB. [3 marks]

P−H=$(-3/5, 6/5)$, whose magnitude is $3/\sqrt{5}$.

The parameter 9/10 lies between zero and one.

Answer: $PH = 3/\sqrt{5}$; H is on the segment.

9. A vector has magnitude 10. Its signed scalar projection onto the positive x-axis is 6. [5 marks]

(a) Find every possible vector. [3 marks]

The x-component is six; $36 + y^2 = 100$.

Thus $y = \pm 8$.

Answer: $(6, 8)$ or $(6, -8)$.

(b) Find the magnitude of its perpendicular component. [2 marks]

The perpendicular component is $(0, \pm 8)$, with length eight.

Answer: 8.

10. A, B and C are the vertices of a non-degenerate triangle, with position vectors a, b and c. Define G by $\mathbf{g} = (\mathbf{a} + \mathbf{b} + \mathbf{c})/3$. Let M be the midpoint of BC. [7 marks]

(a) Show $\overrightarrow{AG} = \frac{2}{3}\overrightarrow{AM}$. [4 marks]

Subtract a from g.

Use OM=$(b + c)/2$ to calculate AM.

Answer: Both sides equal $(\mathbf{b} + \mathbf{c} - 2\mathbf{a})/3$.

(b) State the ratio AG:GM and explain why G is on the median AM. [3 marks]

The remaining third of AM is GM.

The vector scalar multiple proves collinearity.

Answer: $2 : 1$; the positive parameter 2/3 places G on AM.