Vectors in two dimensions - Practice test 5

Name: _____ Date: _____

10 questions | 61 marks | 75 minutes

A more demanding synthesis paper combining unknowns, modelling, projections and general proofs.

A scientific calculator may be used.

1. Let $\mathbf{a} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{b} = -\mathbf{i} + 3\mathbf{j}$. [4 marks]

(a) Find $3\mathbf{a} + 2\mathbf{b}$ in component form. [2 marks]

(b) Find a unit vector in this resultant's direction. [2 marks]

2. A=(k,2), B=(5,m). The displacement BA is $(-3, 4)$. [4 marks]

(a) Find k and m. [2 marks]

(b) Find the exact length of AB and a unit vector from A to B. [2 marks]

3. A non-zero vector v is parallel to (3,4), has magnitude 20 and has a negative x-component. [6 marks]

(a) Find v. [3 marks]

(b) Find its direction from the positive x-axis to 0.1°. [3 marks]

4. Non-zero vectors a and b satisfy $|a| = 2$, $|b| = 3$ and $a \cdot b = 3$. [6 marks]

(a) Find their included angle. [2 marks]

(b) Find $|2\mathbf{a} - \mathbf{b}|$ and the scalar projection of a onto b. [4 marks]

5. A=(-1,2), B=(3,4), C=(6,-2). D completes the parallelogram ABCD in that order. [7 marks]

(a) Find D. [3 marks]

(b) Prove that the parallelogram is a rectangle and find its area. [4 marks]

6. A vector u is perpendicular to (1,2), and its vector projection onto the positive x-axis is (-6,0). [5 marks]

(a) Find u. [3 marks]

(b) Find its scalar projection onto (0,-1). [2 marks]

7. Two adjustable forces are $m(3, 4)$ N and $n(2, -2)$ N, where m,n are positive. Their resultant must be (14,0) N. [7 marks]

(a) Find m and n. [4 marks]

(b) Compare the sum of the force magnitudes with the resultant magnitude. [3 marks]

8. Let u=(1,t) and v=(t,1), where t is real. [7 marks]

(a) Find all t making u and v parallel, and give the corresponding angles. [4 marks]

(b) Find all t making u and v perpendicular. [3 marks]

9. A line through O has direction v=(3,4). P has position p=(-4,3). A point Q on the line has position $t\mathbf{v}$. [7 marks]

(a) Show that $|\overrightarrow{PQ}|^2 = 25 + 25t^2$. [4 marks]

(b) Hence find the closest point on the line to P and the shortest distance, without calculus. [3 marks]

10. In a quadrilateral ABCD, M,N,P,Q are the midpoints of AB,BC,CD,DA respectively. Assume the midpoint quadrilateral is non-degenerate. [8 marks]

(a) Use position vectors to prove that MNPQ is a parallelogram. [4 marks]

(b) Prove that perpendicular diagonals AC and BD make MNPQ a rectangle. [4 marks]