Determinants and inverses of 2×2 matrices

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find $\det\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$. [2 marks]

$3(4) - 2(1) = 10$.

Answer: 10.

2. Find the inverse of $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. [2 marks]

The determinant is 1.

Swap diagonal entries and negate the off-diagonal entries.

Answer: $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$.

3. Find the inverse of $\begin{pmatrix} 2 & 0 \\ 0 & 5 \end{pmatrix}$. [2 marks]

Each diagonal scale is undone by its reciprocal.

The determinant is 10, so the inverse exists.

Answer: $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{5} \end{pmatrix}$.

4. Find t if $\det\begin{pmatrix} t & 1 \\ 2 & 3 \end{pmatrix} = 7$. [2 marks]

$3t - 2 = 7$.

Thus $3t = 9$.

Answer: $t = 3$.

5. Find the inverse of $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. [2 marks]

The determinant is 1.

The inverse reverses the quarter-turn.

Answer: $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

6. Show that $\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$ has no inverse. [2 marks]

$2(2) - 4(1) = 0$.

The inverse formula would require division by zero.

Answer: Its determinant is zero.