Combined transformations and their order

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. First dilate by factor 3, then reflect in the y-axis. Give the matrix. [2 marks]

Multiply the reflection matrix by 3I.

Answer: $\begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix}$.

2. First reflect in $y = x$, then in the x-axis. Describe the result. [2 marks]

The coordinate rule is (x,y) to (y,x), then (y,-x).

Answer: Rotation 90° clockwise.

3. For $A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, find BA. [2 marks]

Use row-column multiplication in the written order.

Answer: $\begin{pmatrix} 0 & -1 \\ 2 & 0 \end{pmatrix}$.

4. For those A and B, find AB and compare. [2 marks]

The off-diagonal entries are different.

Answer: $AB = \begin{pmatrix} 0 & -2 \\ 1 & 0 \end{pmatrix} \neq BA$.

5. Explain why dilation kI commutes with every linear transformation A. [2 marks]

The scalar multiplies each entry of A on either side.

Answer: $A(kI) = (kI)A = kA$.

6. Project $(2, 5)$ onto the x-axis, then rotate 90° counterclockwise. [2 marks]

Projection gives (2,0).

The rotation gives (0,2).

Answer: $(0, 2)$.