Recurrence rules and explicit terms

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. Find the first four terms of $u_n = 2n^2 - 1$, $n \geq 1$. [2 marks]

Substitute $n = 1, 2, 3, 4$ separately.

Answer: $1, 7, 17, 31$.

2. $a_1 = 5$, $a_{n+1} = 2a_n - 1$. Find a_2, a_3, a_4. [2 marks]

$2(5) - 1 = 9$, then $2(9) - 1 = 17$, then $2(17) - 1 = 33$.

Answer: $9, 17, 33$.

3. For $b_n = 30 - 4n$, find b_7. Can a term equal zero for integer $n \geq 1$? [2 marks]

$30 - 28 = 2$. The equation $30 - 4n = 0$ gives $n = 7.5$, not an allowed index.

Answer: $b_7 = 2$; no zero term.

4. $c_0 = 10$, $c_{n+1} = 0.5c_n$. Find c_3. [2 marks]

$c_1 = 5, c_2 = 2.5, c_3 = 1.25$.

Answer: 1.25.

5. Give an explicit rule for $7, 11, 15, 19, \ldots$ with first index 1. [2 marks]

$7 + 4(n - 1) = 4n + 3$.

Answer: $u_n = 4n + 3$.

6. Give two different sequences satisfying $u_{n+1} = u_n + 5$. [3 marks]

Different initial values satisfy the same recurrence.

Answer: For example $1, 6, 11, \ldots$ and $3, 8, 13, \ldots$.