Introduction to proof by induction

6 questions | 16 marks

Equivalent correct methods earn the corresponding marks.

1. Check the base case for $1 + 2 + \cdots + n = n(n + 1)/2$. [2 marks]

The two sides agree for the first permitted integer.

Answer: At $n = 1$, $1 = 1(2)/2$.

2. State the induction hypothesis for the same identity. [2 marks]

This assumes just the arbitrary k case.

Answer: Assume $1 + 2 + \cdots + k = k(k + 1)/2$ for arbitrary integer $k \geq 1$.

3. Complete the induction step for the same identity. [3 marks]

Factor $(k + 1)$ from the two terms.

Answer: $k(k + 1)/2 + (k + 1) = (k + 1)(k + 2)/2$.

4. A student checks $n = 1, 2, 3$ and concludes an identity holds for every positive integer. Identify the missing argument. [2 marks]

Finite checks do not exclude later failure.

Answer: No implication for arbitrary k to $k + 1$ has been established.

5. Prove $2 + 4 + \cdots + 2n = n(n + 1)$ by induction. [4 marks]

Base: $2 = 1(2)$.

Assume $S_k = k(k + 1)$. Add $2(k + 1)$ to get $(k + 1)(k + 2)$; conclude.

Answer: True for all integers $n \geq 1$.

6. Why is an induction step alone insufficient for $1 + 2 + \cdots + n = n(n + 1)/2 + 5$? [3 marks]

A formal same-offset implication can hold without a true starting case.

Answer: The base fails: at $n = 1$ the sides are 1 and 6.