Induction proofs of simple sums and arithmetic/geometric-series results

6 questions | 19 marks

Equivalent correct methods earn the corresponding marks.

1. Prove $3 + 7 + \cdots + (4n - 1) = n(2n + 1)$. [4 marks]

Base: $3 = 1(3)$. Assume $S_k = k(2k + 1)$.

Add $4k + 3$ to get $2k^2 + 5k + 3 = (k + 1)(2k + 3)$; conclude.

Answer: True for integers $n \geq 1$.

2. Prove $1 + 3 + \cdots + 3^{n-1} = (3^n - 1)/2$. [4 marks]

Base: $1 = (3 - 1)/2$. Assume $S_k = (3^k - 1)/2$.

Add 3^k to get $(3^{k+1} - 1)/2$; conclude.

Answer: True for integers $n \geq 1$.

3. State the next term and required restriction in a proof of $a + ar + \cdots + ar^{n-1} = a(1 - r^n)/(1 - r)$. [2 marks]

The denominator must not be zero.

Answer: Next term ar^k; restriction $r \neq 1$.

4. Simplify $a(1 - r^k)/(1 - r) + ar^k$ to the next-case formula. [3 marks]

The numerator becomes $a[1 - r^k + r^k(1 - r)] = a(1 - r^{k+1})$.

Answer: $a(1 - r^{k+1})/(1 - r)$.

5. Use an arithmetic sum to find $5 + 8 + 11 + \cdots + 62$. [2 marks]

$62 = 5 + 3(n - 1)$ gives $n = 20$; $20(5 + 62)/2 = 670$.

Answer: 670.

6. Prove $2 + 6 + \cdots + 2 \cdot 3^{n-1} = 3^n - 1$. [4 marks]

Base: $2 = 3 - 1$. Assume $S_k = 3^k - 1$.

Add $2 \cdot 3^k$ to obtain $3^{k+1} - 1$; conclude.

Answer: True for integers $n \geq 1$.