Sequences and series - Practice test 1

10 questions | 53 marks | 70 minutes

Terms, sums and first model choices.

Equivalent correct methods earn the corresponding marks.

1. $u_1 = 4$ and $u_{n+1} = u_n + 7$. [5 marks]

(a) Find u_2, u_3, u_4. [2 marks]

Add 7 at each step.

Answer: $11, 18, 25$.

(b) Give an explicit rule and find u_{30}. [3 marks]

$4 + 7(n-1) = 7n - 3$; substitute 30.

Answer: $u_n = 7n - 3$; $u_{30} = 207$.

2. An arithmetic sequence has $u_3 = 16, u_9 = 46$. [5 marks]

(a) Find its first term and difference. [2 marks]

$6d = 30$; $a + 2d = 16$.

Answer: $a = 6, d = 5$.

(b) Decide whether 121 is a term. [3 marks]

$u_n = 5n + 1$; $5n + 1 = 121$ gives $n = 24$.

Answer: Yes, term 24.

3. An auditorium has 8 seats in its first row and 3 extra seats in each next row. [5 marks]

(a) Find the seats in row 12. [2 marks]

$8 + 11(3) = 41$.

Answer: 41.

(b) Find the total seats in 12 rows. [3 marks]

$S_{12} = 12(8 + 41)/2 = 294$.

Answer: 294.

4. The geometric sequence begins $8, -24, 72, \ldots$. [5 marks]

(a) Find its ratio and seventh term. [2 marks]

$r = -24/8 = -3$; $8(-3)^6 = 5832$.

Answer: $r = -3, u_7 = 5832$.

(b) Find the sum of its first six terms. [3 marks]

$S_6 = 8[1 - (-3)^6]/[1 - (-3)] = -1456$.

Answer: -1456.

5. A geometric sequence has $u_1 = 5, u_3 = 20$. [5 marks]

(a) Find all possible real ratios. [2 marks]

$5r^2 = 20$.

Answer: $r = 2$ or -2.

(b) For each ratio, find u_2 and u_4. [3 marks]

$u_2 = 5r$ and $u_4 = 5r^3$.

Answer: $10, 40$ if $r = 2$; $-10, -40$ if $r = -2$.

6. A ball falls 3 m and rebounds to one-third of the preceding height after every bounce. [6 marks]

(a) Find the first two rebound heights. [2 marks]

Multiply the original height by $1/3$ and then again.

Answer: 1 m and $1/3$ m.

(b) Find its ideal total vertical distance. [4 marks]

$3 + 2[1/(1 - 1/3)] = 6$. State that the model allows infinitely many ever-smaller bounces.

Answer: 6 m.

7. A flushing process starts with 10 g of salt. Each step retains 80%, then adds 6 g. [6 marks]

(a) Write a recurrence and calculate the amount after three steps. [3 marks]

$M_1 = 14, M_2 = 17.2, M_3 = 19.76$.

Answer: $M_{n+1} = 0.8M_n + 6$, $M_0 = 10$; $M_3 = 19.76$ g.

(b) Find and interpret the equilibrium. [3 marks]

$L = 0.8L + 6$; the deviation factor 0.8 has magnitude below 1.

Answer: 30 g; the model approaches this value.

8. AUD 300 is deposited at the end of each year for four years at 6% annual interest. [5 marks]

(a) Write the four contributions to the final balance. [2 marks]

The latest payment earns no interest before the observation time.

Answer: $300(1.06)^3, 300(1.06)^2, 300(1.06), 300$.

(b) Calculate the balance immediately after the fourth deposit. [3 marks]

$300[(1.06)^4 - 1]/0.06 = 1312.3848$.

Answer: AUD 1312.38 to the nearest cent.

9. Consider $3 + 5 + \cdots + (2n + 1) = n(n + 2)$. [6 marks]

(a) Check the base case and state a valid induction hypothesis. [2 marks]

The hypothesis concerns the k case only.

Answer: At $n = 1$: $3 = 1(3)$. Assume $S_k = k(k + 2)$ for arbitrary integer $k \geq 1$.

(b) Complete the induction step and conclusion. [4 marks]

Add the next term and factor.

Answer: $S_{k+1} = k(k + 2) + 2k + 3 = (k + 1)(k + 3)$; hence true for all integers $n \geq 1$.

10. A student says: “If sequence terms approach zero, their sum must approach zero.” [5 marks]

(a) Use $1, 1/2, 1/4, \ldots$ to refute the statement. [2 marks]

$1/(1 - 1/2) = 2$.

Answer: The terms approach zero but the sum approaches 2.

(b) Explain the distinction between a term and a partial sum. [3 marks]

The two sequences can have different limits.

Answer: A term is one entry u_n; a partial sum adds entries from the start through index n.